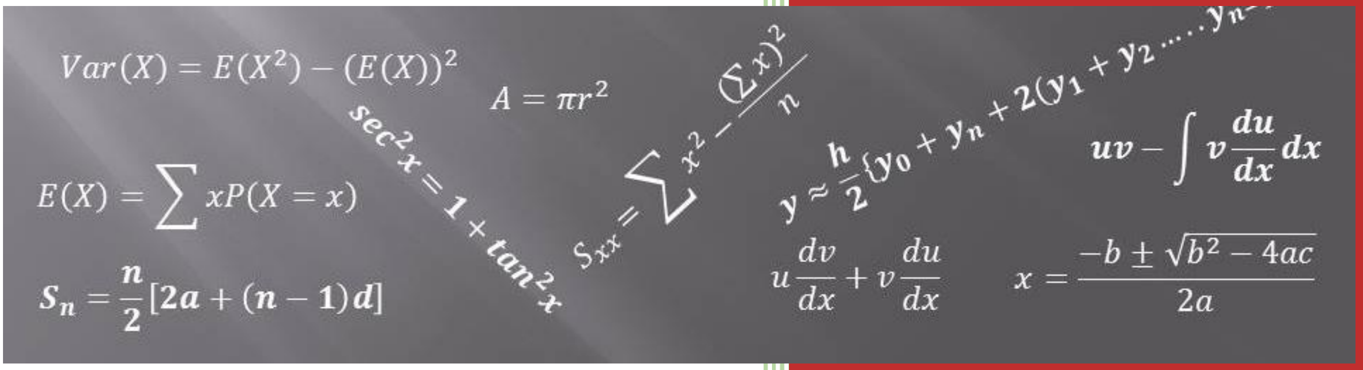
MathsGeeks

'Everything You Need to Know'
A Level – Edexcel – C3



September 2013

MathsGeeks

Trigonometry: Learn (or learn how to derive)

$\sec x = \frac{1}{\cos x}$ $cosec\ x = \frac{1}{\sin x}$ $\cot x = \frac{1}{\tan x}$

Double angle formulas $\sin 2\theta \equiv 2sin\theta cos\theta$

$$\cos 2\theta \equiv cos^2\theta - sin^2\theta$$

$$\cos 2\theta \equiv 2cos^2\theta - 1$$

$$\cos 2\theta \equiv 1 - 2sin^2\theta$$

$$\tan 2\theta \equiv \frac{2\tan\theta}{1-tan^2\theta}$$

(Note that $cos\ 2\theta$ can be changed into either cos or sin so pick the one most appropriate to the question).

Also learn $1 +\ tan^2x = sec^2x$

$$cot^2x + 1 = cosec^2x$$

When carrying out a proof always pick the most complicated side, as it is easier to simplify than expand. Also learn to use formulas for $\sin(A+B)$ and $\cos(A+B)$in the Formula Book (FB).

Problems in the form: If $y = \sqrt{3}cosx + sinx$ write in the form $y\ =\ Rsin(x\ +\ \alpha)$, then expand out to get $y = Rsinxcos\alpha + Rcosxsin\alpha$ then compare this with original equation to see that $Rcos\alpha = 1$ and $Rsin\alpha = \sqrt{3}$ then solve simultaneously to get R and α e.g. $R^2 = 1^2 + \sqrt{3}^2 = 4$ and $R = 2$ and $\frac{\sin\alpha}{\cos\alpha} = tan\alpha = \frac{\sqrt{3}}{1}$ and $\alpha = tan^{-1}\sqrt{3}$.

Functions: A function is a mapping between two variables, usually x - the independent variable, and y the dependent variable. The language of functions is as follows:

- **Domain:** The set of values taken by x (the independent variable)
- **Range:** The resulting values of y arising is the Range.
- **Inverse:** Only exists for a one to one mapping. e.g. if they ask find $f^{-1}x$ where $f: x \mapsto 1 - 2x^3$ then let replace $f(x)$ with x and x with y and then solve for y. e.g. $x = 1 - 2y^3$ and $y = \sqrt[3]{\frac{1}{2}(1-x)} = f^{-1}(x)$.
- **Composite:** if you are asked for gf that means apply $f(x)$ and then $g(x)$. e.g. if $f(x) = 1 - 2x^3$ and $g(x) = \frac{3}{x} - 4$ then $gf(x) = \frac{3}{(1-2x^3)} - 4$. Note that $gf(x) \neq fg(x)$ generally.
- **Roots:** If you are asked to show that a function has a root between say [-1, 2] then first substitute in -1 and then 2 and show that there is a change of sign. Make sure you right a sentence to explain that a change of sign means that there is a root between those two values.
- **Partial Fractions:** For ‘show’ questions usually you find a common denominator and then expand out. Either compare coefficients of terms in x^n or substitute in particular values of x

to find unknowns. e.g. if $f(x) = \frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{x+4}$ then $f(x) = \frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$ and simplify to $f(x) = \frac{x+4}{(2x-1)(x+4)}$ therefore $f(x) = \frac{1}{(2x-1)}$.

- **Quadratics:** To show that a quadratic is always > 0 then complete the square. e.g. $x^2 + x + 1 > 0$ then $(x+\frac{1}{2})^2 + \frac{3}{4} > 0$ but as the first term is squared it is always > 0 and obviously $\frac{3}{4} > 0$. Make sure you always include a sentence explaining this.

Iteration: Best explained by example. If given $x_{n+1} = ln(x_n + 2) + 1, \quad x_0 = 2.5$ and asked to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places. Then you simply put x_0 into the formula to find $x_1 = \ln(2.5+2) + 1 = 2.50408$ and then to find x_2 you simply sub in x_1 e.g. $x_2 = \ln(2.50408 + 2) + 1 = 2.50498$ and so on.

If asked to show say 2.505 is a root of $f(x) = ln\,(x+2) - x + 1$ then pick appropriate limits that would round up or down to 2.505 show there is a change of sign. i.e. do $f(2.5045)$ and $f(2.5055)$.

Exponentials and Logarithms: Remember $e^{ln\,8} = 8$. Usually to solve the equations involving e you have to take logs of both sides e.g. if $2 = e^{1.5t}$ then $ln\,2 = 1.5t$ and $t = \frac{2}{3}ln2$.

Further Differentiation: Learn or remember from C1 plus learn how to use.

	$\boldsymbol{y = f(x)}$	$\boldsymbol{\frac{dy}{dx} = f'(x)}$
Learn	e^x	e^x
Learn	$\ln x$	$\frac{1}{x}$
Learn	$\sin nx$	$n\cos nx$
Learn	$\cos nx$	$-n\sin nx$
Learn	$(ax+b)^n$	$na(ax+b)^{n-1}$
Learn	$u(x)v(x)$	$u\frac{dv}{dx} + v\frac{du}{dx}$
In FB	$\frac{u(x)}{v(x)}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
In FB	$\tan x$	$sec^2 x$
In FB	$\cot x$	$-cosec^2 x$
In FB	$\sec x$	$\sec x \tan x$
In FB	$\mathrm{cosec}\, x$	$-cosec\ x \cot x$

Remember that turning points occur at $\frac{dy}{dx} = 0$ and that if $y = mx + c$ of a tangent then $m = \frac{dy}{dx}$ and for a normal $\frac{1}{m} = -\frac{dy}{dx}$. If given a function where $x = f(y)$ to find $\frac{dy}{dx}$ do $\frac{dx}{dy}$ and then do $\frac{1}{dx/dy}$.