

## 'Everything You Need to Know' A Level – Edexcel – C4

$Var(X) = E(X^2) - (E(X))^2$

$E(X) = \sum xP(X = x)$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$A = \pi r^2$

$\sec^2 x = 1 + \tan^2 x$

$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$

$y \approx \frac{h}{2}(y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1}))$

$uv - \int v \frac{du}{dx} dx$

$u \frac{dv}{dx} + v \frac{du}{dx}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Further Binomial Expansion:** Make sure it starts with a 1 e.g. for  $(2 - x)^{-2} = 2^{-2}(1 - \frac{x}{2})^{-2}$  then use

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}.$$

**Parametric Equations:** to find  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$  e.g.  $x = 7\cos t - \cos 7t$ ,  $y = 7\sin t - \sin 7t$  then

$$\frac{dx}{dt} = -7\sin t + 7\sin 7t \text{ and } \frac{dy}{dt} = 7\cos t - 7\cos 7t \text{ so } \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}. \text{ Recall that the gradient at any point is } \frac{dy}{dx} \text{ and the gradient of the normal} = -\frac{dx}{dy}.$$

**Partial Fractions:** There are three methods only 1) Linear expand as e.g.  $\frac{2x-1}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3} =$

$$\frac{A(2x-3)+B(x-1)}{(x-1)(2x-3)} \text{ and compare coefficients } x, x^2 \text{ and numbers of top line the top line e.g. } 2 = 2A + B$$

and  $-1 = -3A - B$  then solve simultaneously. 2) Repeated factor in the denominator e.g.

$$\frac{x^2-2x-1}{(x+1)(x-1)^2} \text{ solve as follows } \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \text{ and solve as above i.e. } x^2 - 2x - 1 = A(x-1)^2 +$$

$$B(x+1)(x-1) + C(x+1). \text{ 3) When the top is bigger than the bottom } \frac{2x^2+8x+7}{x^2+5x+6} \text{ divide the}$$

$$\text{denominator into the numerator by long division and you are left with } 2 - \frac{2x+5}{x^2+5x+6}. \text{ Factorise the}$$

bottom and solve as method 1).

## Further Differentiation

**Implicit Differentiation:** Recognise these as questions that ask for  $\frac{dy}{dx}$  from functions of  $x$

AND  $y$  e.g. (1) Find  $\frac{dy}{dx}$  for  $x^3 + xy^2 - y^3 = 5$  e.g. (2) Find  $\frac{dy}{dx}$  for  $y = xe^y$ . Differentiate each

term in turn e.g. (1)  $\frac{d}{dx}(x^3) + \frac{d}{dx}(xy^2) + \frac{d}{dx}(-y^3) = \frac{d}{dx}(5)$ . Don't forget to differentiate

any products using the product rule and remember that  $\frac{d}{dx}(f(y)) = \frac{d}{dy}f(y) \frac{dy}{dx}$ . So e.g.(1)

is  $3x^2 + y^2 + x \cdot 2y \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$ . Then rearrange to make  $\frac{dy}{dx}$  the subject so  $\frac{dy}{dx} =$

$\frac{-3x^2 - y^2}{2yx - 3y^2}$ . e.g. (2)  $\frac{d}{dx}(y) = \frac{d}{dx}(xe^y)$ ,  $\frac{dy}{dx} = e^y + xe^y \frac{dy}{dx}$  and rearrange  $\frac{dy}{dx} = \frac{e^y}{1 - xe^y}$ . (Remember

this can be used to find a gradient of a tangent/normal etc.).

**Separating variables:** Recognise these questions as having  $x, y$  and  $\frac{dy}{dx}$  e.g. (1)  $\frac{1}{x} \frac{dy}{dx} =$

$\frac{2y}{x^2+1}$ , e.g. (2)  $2 \frac{dy}{dx} = \frac{\cos x}{y}$ . Separate so that all the  $y$ 's are on the side  $dy$  and all the  $x$ 's are

on the side  $dx$  and integrate both sides. e.g. (1)  $\int \frac{1}{y} dy = \int \frac{2x}{x^2+1} dx$  therefore  $\ln|y| =$

$\ln|x^2 + 1| + A$ . e.g.(2)  $\int 2y dy = \int \cos x dx$  therefore  $y^2 = \sin x + A$ .

**Rate Questions:** the word 'rate' means differentiate with respect to time  $t$ . e.g. the rate of change of volume would be  $\frac{dV}{dt}$ . Naturally occurring differential equations occur for

exponential growth or decay e.g. growth of yeast cells given by  $\frac{dn}{dt} = kn$  and solving by

separating variables (above) gives  $n = n_0 e^{kt}$ . Common question type e.g. A right cylinder is expanding as heated. After  $t$  seconds the radius is  $x$  cm and the length is  $5x$  cm. The cross-

sectional area of the cylinder is increasing at the constant rate of  $0.032 \text{ cm}^2$ . a) Find  $\frac{dx}{dt}$  when the radius of the rod is 2cm. b) Find the rate of increase of the volume when  $x = 2$ . a) In the question we are given  $\frac{dA}{dt} = 0.032$  therefore  $\frac{dx}{dt} = \frac{dA}{dt} \times \frac{dx}{dA}$  and we need a relationship between  $x$  and  $A$ . But  $A$  is the cross sectional area of a cylinder so  $A = \pi x^2$  so differentiating gives  $\frac{dA}{dx} = 2\pi x$ . Therefore  $\frac{dx}{dt} = 0.032 \times \frac{1}{2\pi x} = 0.0025 \text{ cms}^{-1}$ . b) The rate of increase in volume is  $\frac{dV}{dt}$  and so using part a) we get  $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$  and we need a relationship between  $V$  and  $x$  which for a cylinder  $V = \pi x^2(5x) = 5\pi x^3$  differentiating to give  $\frac{dV}{dx} = 15\pi x^2$  and therefore  $\frac{dV}{dt} = 0.0025 \times 15\pi \times 4 = 0.48 \text{ cm}^3 \text{ s}^{-1}$ .

## Vectors

**Equation of a line** is given by  $r = x_1i + y_1j + z_1k + t(x_2i + y_2j + z_2k)$  where  $x_1i + y_1j + z_1k$  is any point on the line,  $x_2i + y_2j + z_2k$  is the direction of the line and  $t$  is a variable that has specific value for each point on the line. e.g. find a vector equation for the line going through A and B where  $A = 2i + 6j - k$  and  $B = 3i + 4j + k$  then  $AB = b - a = (3 - 2)i + (4 - 6)j + (-1 - 1)k = i - 2j - 2k$  and therefore a vector equation of the line through AB can be written as  $r = (2i + 6j - k) + t(i - 2j + 2k)$ .

**Intersection of lines:** given two vector equations of lines and asked to show they meet (or not) you must resolve the parts  $i, j$  and  $k$  independently because for intersection the  $i, j$  and  $k$  values must ALL be the equal. e.g. if  $r_1 = (-9i + 10k) + s(2i + j - k)$  and  $r_2 = (3i + j + 17k) + t(3i - j + 5k)$  then  $-9 + 2s = 3 + 3t$  and  $s = 1 - t$  solve these two simultaneously and show these values also work for  $k$  i.e.  $10 - s = 17 + 5t$ .

**Length of a line** the distance between two points is the length of the vector so for AB above the length of AB is  $|AB|$  is given by  $\sqrt{1^2 + (-2)^2 + (-2)^2}$ .

**Angles between 2 lines:** if asked to find the angle between two lines then do  $\cos\theta = \frac{r_1 \cdot r_2}{|r_1||r_2|}$  where  $r_1$  and  $r_2$  are the direction vectors of the lines e.g for A and B above  $\cos\theta = \frac{(2i+6j-k) \cdot (3i+4j+k)}{\sqrt{4+36+1}\sqrt{9+16+1}} = \frac{6+24-1}{\sqrt{41}\sqrt{26}}$  and solve.

**Integration Techniques:** recall the trapezium rule from C1 and apply to functions including  $e^x$  and recall exact integration techniques from C2. Other new techniques are summarised on the next page. Recall area under the curve is given by  $\text{Area} = \pi \int y \, dx$ . Learn that the volume of revolution around the x-axis is  $\text{Vol} = \pi \int y^2 dx$  and around y-axis is  $\text{Vol} = \pi \int x^2 dy$ . Learn to use formulas in the book and don't forget to use the differentiation formulas in reverse.

What it looks like	How to do it	Example
<b>Recognition:</b> Use when the function has 'almost' the differential included. e.g.(1) $\int \sin x \cos^3 x dx$ e.g. (2) $\int x e^{x^2} dx$	Look at it backwards: differentiate the approximate answer. (1) $\frac{d}{dx}(\cos^4 x) = 4(-\sin x)\cos^3 x$ (2) $\frac{d}{dx}(e^{x^2}) = 2x e^{x^2}$	And then rearrange to give: (1) $\int \sin x \cos^3 x dx = -\frac{1}{4}\cos^4 x + K$ (2) $\int x e^{x^2} dx = \frac{1}{2}e^{x^2} + K$
<b>Substitution.</b> e.g. (1) $\int_0^1 x^2 \sqrt{x^3 + 1} dx$ e.g. (2) $\int_0^{\pi/2} \cos x \sin^3 x dx$ (note they usually suggest the substitution)	e.g. (1) Substitute is $u = x^3 + 1$ Then differentiate both sides $\dots du = 3x^2 dx$ Change the limits When $x = 0$ then $u = 1$ When $x = 1$ then $u = 2$ e.g.(2) Substitute is $u = \sin x$ $\dots du = \cos x dx$ When $x = \pi/2$ then $u = 1$ . When $x = 0$ then $u = 0$ .	e.g. (1) $\int_0^1 x^2 \sqrt{x^3 + 1} dx = \frac{1}{3} \int_1^2 \sqrt{u} du$ $= \frac{1}{3} \left[ \frac{2}{3} u^{3/2} \right]_1^2 = \frac{2}{9} (2\sqrt{2} - 1)$ e.g.(2) $\int_0^{\pi/2} \cos x \sin^3 x dx = \int_0^1 u^3 du$ $= \left[ \frac{u^4}{4} \right]_0^1 = \frac{1}{4}$
<b>Partial fractions</b> e.g. $\int \frac{1}{(x-1)(x-3)} dx$ (with a quadratic in the denominator)	Separate into partial fraction as shown earlier. $\frac{1}{(x-1)(x-3)} = \frac{A(x-3) + B(x-1)}{(x-1)(x-3)}$	$\int -\frac{1}{2(x-1)} + \frac{1}{2(x-3)} dx$ $= -\frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x-3} dx$ $= -\frac{1}{2} \ln x-1  + \frac{1}{2} \ln x-3  + K$ (then simplify this answer!)
$\int c e^{ax+b} dx$	$= \frac{c}{a} e^{ax+b} + K$	e.g. $\int 3e^{2x+4} dx = \frac{3}{2} e^{2x+4} + K$
$\int \ln x dx$	Use product rule with $u = \ln x$ ( $\frac{du}{dx} = \frac{1}{x}$ ) and $\frac{dv}{dx} = 1$ ( $v = x$ ).	$\int \ln x dx = x \ln x - \int x \frac{1}{x} dx$ $= x \ln x - x + K$
$\int \frac{1}{ax+b} dx$	$= \frac{1}{a} \ln ax+b  + K$	e.g. $\int \frac{1}{4-3x} dx = -\frac{1}{3} \ln 4-3x  + K$
$\int (ax+b)^n dx$	$= \frac{1}{a(n+1)} (ax+b)^{n+1}$	e.g. $\int (2x+3)^5 dx = \frac{1}{12} (2x+3)^6 + K$
$\int \frac{1}{(1+2x)^2} dx$	Convert to $\int (1+2x)^{-2} dx$	$= -\frac{1}{2} (1+2x)^{-1} + K$
<b>Integration by parts:</b> $\int u \frac{dv}{dx} dx$ e.g. $\int x e^{2x} dx$	$= uv - \int v \frac{du}{dx} dx$ (This is in the formula book) Let $u = x$ and $\frac{dv}{dx} = e^{2x}$ then $\frac{du}{dx} = 1$ and $v = \frac{1}{2} e^{2x}$ .	$\int x e^{2x} dx = \frac{x e^{2x}}{2} - \int \frac{1}{2} e^{2x} dx$ $= \frac{x e^{2x}}{2} - \frac{1}{4} e^{2x} + K$
<b>Trigonometry</b> $\int c \cos(ax+b) dx$	$= \frac{c}{a} \sin(ax+b) + K$	e.g. $\int 5 \cos(4-3x) dx = -\frac{5}{3} \sin(4-3x) + K$ (note the $a$ and $b$ are the other way around!)

$\int c \sin(ax + b) dx$	$= -\frac{c}{a} \cos(ax + b) + K$	$\int 7 \sin(8x + 9) dx = -\frac{7}{8} \cos(8x + 9) + K$
$\int \sin^2 x dx$ or $\int \cos^2 x dx$	Convert to $\cos 2x$ using $\cos^2 x - \sin^2 x = \cos 2x$ and $\cos^2 x + \sin^2 x = 1$ . So $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	e.g. $\int \sin^2 x dx = \frac{1}{2} \int (1 - \cos 2x) dx$ $= \frac{x}{2} - \frac{1}{4} \sin 2x + K$ . (note: this comes up on almost every paper!)
e.g. $\int \cos^5 x dx$ or any odd power of cos or sin.	Convert using $\cos^2 x = 1 - \sin^2 x$ $\cos^5 x = (1 - \sin^2 x)^2 \cos x$	Expand to get $\int (1 - 2\sin^2 x + \sin^4 x) \cos x dx$ $= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + K$
e.g. $\int \sin^3 x \cos^2 x dx$ or any multiple powers of sin and cos.	Convert using $\cos^2 x + \sin^2 x = 1$ to give a single cos or sin.	e.g. $= \int \sin x (1 - \cos^2 x) \cos^2 x dx$ $= \int \sin x \cos^2 x - \sin x \cos^4 x dx$ $= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + K$
e.g. $\int \tan^5 x dx$ or any power of $\tan x$	Use $\tan^2 x = \sec^2 x - 1$	e.g. $= \int \tan^3 x (\sec^2 x - 1) dx$ $= \int \sec^2 x \tan^3 x - \int \tan x (\sec^2 x - 1) dx$ $= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \ln \sec x + K$

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