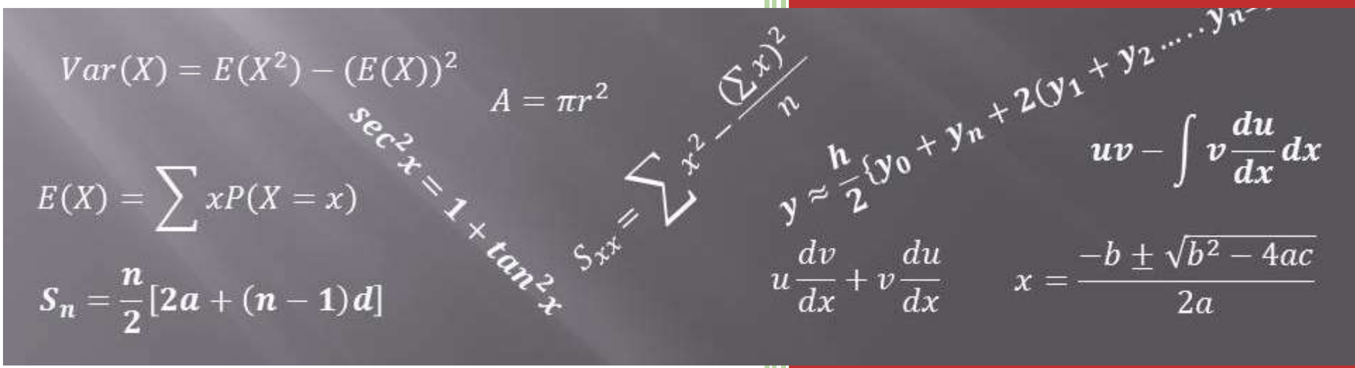
MathsGeeks

'Everything You Need to Know'
A Level – Edexcel – C4



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Further Binomial Expansion: Make sure it starts with a 1 e.g. for $(2-x)^{-2} = 2^{-2}(1-\frac{x}{2})^{-2}$ then use $(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$.

Parametric Equations: to find $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ e.g. $x = 7cost - cos7t, \quad y = 7sint - sin7t$ then $\frac{dx}{dt} = -7sint + 7sin7t$ and $\frac{dy}{dt} = 7cost - 7cos7t$ so $\frac{dy}{dx} = \frac{7cost-7cos7t}{-7sint+7sin7t}$. Recall that the gradient at any point is $\frac{dy}{dx}$ and the gradient of the normal = $-\frac{dx}{dy}$.

Partial Fractions: There are three methods only 1) Linear expand as e.g. $\frac{2x-1}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3} = \frac{A(2x-3)+B(x-1)}{(x-1)(2x-3)}$ and compare coefficients x, x^2 and numbers of top line the top line e.g. $2 = 2A + B$ and $-1 = -3A - B$ then solve simultaneously. 2) Repeated factor in the denominator e.g. $\frac{x^2-2x-1}{(x+1)(x-1)^2}$ solve as follows $\frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ and solve as above i.e. $x^2 - 2x - 1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$. 3) When the top is bigger than the bottom $\frac{2x^2+8x+7}{x^2+5x+6}$ divide the denominator into the numerator by long division and you are left with. $2 - \frac{2x+5}{x^2+5x+6}$. Factorise the bottom and solve as method 1).

Further Differentiation

Implicit Differentiation: Recognise these as questions that ask for $\frac{dy}{dx}$ from functions of x AND y e.g. (1) Find $\frac{dy}{dx}$ for $x^3 + xy^2 - y^3 = 5$ e.g. (2) Find $\frac{dy}{dx}$ for $y = xe^y$. Differentiate each term in turn e.g. (1) $\frac{d}{dx}(x^3) + \frac{d}{dx}(xy^2) + \frac{d}{dx}(-y^3) = \frac{d}{dx}(5)$. Don't forget to differentiate any products using the product rule and remember that $\frac{d}{dx}(f(y)) = \frac{d}{dy}f(y)\frac{dy}{dx}$. So e.g.(1) is $3x^2 + y^2 + x.2y\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$. Then rearrange to make $\frac{dy}{dx}$ the subject so $\frac{dy}{dx} = \frac{-3x^2-y^2}{2yx-3y^2}$. e.g. (2) $\frac{d}{dx}(y) = \frac{d}{dx}(xe^y)$, $\frac{dy}{dx} = e^y + xe^y\frac{dy}{dx}$ and rearrange $\frac{dy}{dx} = \frac{e^y}{1-xe^y}$. (Remember this can be used to find a gradient of a tangent/normal etc.).

Separating variables: Recognise these questions as having x, y and $\frac{dy}{dx}$ e.g. (1) $\frac{1}{x}\frac{dy}{dx} = \frac{2y}{x^2+1}$, e.g. (2) $2\frac{dy}{dx} = \frac{\cos x}{y}$. Separate so that all the $y's$ are on the side dy and all the $x's$ are on the side dx and integrate both sides. e.g. (1) $\int \frac{1}{y}dy = \int \frac{2x}{x^2+1}dx$ therefore $ln|y| = ln|x^2+1| + A$. e.g.(2) $\int 2y\, dy = \int \cos x\, dx$ therefore $y^2 = sinx + A$.

Rate Questions: the word 'rate' means differentiate with respect to time t. e.g. the rate of change of volume would be $\frac{dV}{dt}$. Naturally occurring differential equations occur for exponential growth or decay e.g. growth of yeast cells given by $\frac{dn}{dt} = kn$ and solving by separating variables (above) gives $n = n_0e^{kt}$. Common question type e.g. A right cylinder is expanding as heated. After t seconds the radius is x cm and the length is $5x$ cm. The cross-sectional area of the cylinder is increasing at the constant rate of 0.032 cm^2. a) Find $\frac{dx}{dt}$ when the radius of the rod is 2cm. b) Find the rate of increase of the volume when $x = 2$. a) In the question we are given $\frac{dA}{dt} = 0.032$ therefore $\frac{dx}{dt} = \frac{dA}{dt} \times \frac{dx}{dA}$ and we need a relationship between x and A. But A is the cross sectional area of a cylinder so $A = \pi x^2$ so differentiating gives $\frac{dA}{dx} = 2\pi x$. Therefore $\frac{dx}{dt} = 0.032 \times \frac{1}{2\pi x} = 0.0025\ cms^{-1}$. b) The rate of increase in volume is $\frac{dV}{dt}$ and so using part a) we get $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ and we need a relationship between V and x which for a cylinder $V = \pi x^2(5x) = 5\pi x^3$differentiating to give $\frac{dV}{dx} = 15\pi x^2$ and therefore $\frac{dV}{dt} = 0.0025 \times 15\pi \times 4 = 0.48\ cm^3s^{-1}$.

Vectors

Equation of a line is given by $r = x_1i + y_1j + z_1k + t(x_2i + y_2j + z_2k)$ where $x_1i + y_1j + z_1k$ is any point on the line, $x_2i + y_2j + z_2k$ is the direction of the line and t is a variable that has specific value for each point on the line. e.g. find a vector equation for the line going through A and B where $A = 2i + 6j - k$ and $B = 3i + 4j + k$ then $AB = b - a = (3-2)i + (4-6)j + (-1-1)k = i - 2j - 2k$ and therefore a vector equation of the line through AB can be written as $r = (2i + 6j - k) + t(i - 2j + 2k)$.

Intersection of lines: given two vector equations of lines and asked to show they meet (or not) you must resolve the parts i, j and k independently because for intersection the i, j and k values must ALL be the equal. e.g. if $r_1 = (-9i + 10k) + s(2i + j - k)$ and $r_2 = (3i + j + 17k) + t(3i - j + 5k)$ then $-9 + 2s = 3 + 3t$ and $s = 1 - t$ solve these two simultaneously and show these values also work for k i.e. $10 - s = 17 + 5t$.

Length of a line the distance between two points is the length of the vector so for AB above the length of AB is $|AB|$is given by $\sqrt{1^2 + (-2)^2 + (-2)^2}$.

Angles between 2 lines: if asked to find the angle between two lines then do $cos\theta = \frac{r_1.r_2}{|r_1||r_2|}$ where r_1 and r_2 are the direction vectors of the lines e.g for A and B above $cos\theta = \frac{(2i+6j-k).(3i+4j+k)}{\sqrt{4+36+1}\sqrt{9+16+1}} = \frac{6+24-1}{\sqrt{41}\sqrt{26}}$ and solve.

Integration Techniques: recall the trapezium rule from C1 and apply to functions including e^x and recall exact integration techniques from C2. Other new techniques are summarised on the next page. Recall area under the curve is given by Area = $\pi \int y\, dx$. Learn that the volume of revolution around the x-axis is Vol=$\pi \int y^2 dx$ and around y-axis is Vol=$\pi \int x^2 dy$. Learn to use formulas in the book and don't forget to use the differentiation formulas in reverse.

What it looks like	How to do it	Example
Recognition: Use when the function has 'almost' the differential included. e.g.(1)$\int sinx\, cos^3xdx$ e.g. (2)$\int xe^{x^2}dx$	Look at it backwards: differentiate the approximate answer. (1) $\frac{d}{dx}(cos^4x) = 4(-sinx)cos^3x$ (2) $\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}$	And then rearrange to give: (1)$\int sinx\, cos^3xdx = -\frac{1}{4}cos^4x + K$ (2)$\int xe^{x^2}dx = \frac{1}{2}e^{x^2} + K$
Substitution. e.g. (1) $\int_0^1 x^2\sqrt{(x^3+1)}dx$ e.g. (2) $\int_0^{\pi/2} \cos x sin^3xdx$ (note they usually suggest the substitution)	e.g. (1) Substitute is $u = x^3 + 1$ Then differentiate both sides $\ldots du =..3x^2dx$ Change the limits When $x = 0$ then $u = 1$ When $x = 1$ then $u = 2$ e.g.(2) Substitute is $u = sinx$ $\ldots du =..\cos x\, dx$ When $x = \pi/2$ then $u = 1$. When $x = 0$ then $u = 0$.	e.g. (1) $\int_0^1 x^2\sqrt{(x^3+1)}dx = \frac{1}{3}\int_1^2 \sqrt{u}\,du$ $= \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_1^2 = \frac{2}{9}(2\sqrt{2}-1)$ e.g.(2) $\int_0^{\frac{\pi}{2}} \cos x sin^3xdx = \int_0^1 u^3du$ $= \left[\frac{u^4}{4}\right]_0^1 = \frac{1}{4}$
Partial fractions e.g. $\int \frac{1}{(x-1)(x-3)}$ dx (with a quadratic in the denominator)	Separate into partial fraction as shown earlier. $\frac{1}{(x-1)(x-3)} = \frac{A(x-3)+B(x-1)}{(x-1)(x-3)}$	$\int -\frac{1}{2(x-1)} + \frac{1}{2(x-3)}dx$ $= -\frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{x-3}dx$ $= -\frac{1}{2}ln\|x-1\| + \frac{1}{2}ln\|x-3\| + K$ (then simplify this answer!)
$\int ce^{ax+b}dx$	$= \frac{c}{a}e^{ax+b} + K$	e.g. $\int 3e^{2x+4}dx = \frac{3}{2}e^{2x+4} + K$
$\int \ln x\, dx$	Use product rule with $u = \ln x$ ($\frac{du}{dx} = \frac{1}{x}$) and $\frac{dv}{dx} = 1$ $(v = x)$.	$\int \ln x\, dx = xlnx - \int x\frac{1}{x}dx$ $= xlnx - x + K$
$\int \frac{1}{ax+b}dx$	$= \frac{1}{a}ln\|ax+b\| + K$	e.g. $\int \frac{1}{4-3x}dx = -\frac{1}{3}ln\|4-3x\| + K$
$\int (ax+b)^n dx$	$= \frac{1}{a(n+1)}(ax+b)^{n+1}$	e.g. $\int (2x+3)^5dx = \frac{1}{12}(2x+3)^6 + K$
$\int \frac{1}{(1+2x)^2}dx$	Convert to $\int (1+2x)^{-2}dx$	$= -\frac{1}{2}(1+2x)^{-1} + K$
Integration by parts: $\int u\frac{dv}{dx}dx$ e.g. $\int xe^{2x}dx$	$= uv - \int v\frac{du}{dx}dx$ (This is in the formula book) Let $u = x$ and $\frac{dv}{dx} = e^{2x}$ then $\frac{du}{dx} =$ 1 and $v = \frac{1}{2}e^{2x}$.	$\int xe^{2x}dx = \frac{xe^{2x}}{2} - \int \frac{1}{2}e^{2x}dx$ $= \frac{xe^{2x}}{2} - \frac{1}{4}e^{2x} + K$
Trigonometry $\int c\cos(ax+b)\, dx$	$= \frac{c}{a}\sin(ax+b) + K$	e.g. $\int 5\cos(4-3x)dx = -\frac{5}{3}\sin(4-3x) + K$ (note the *a* and *b* are the other way around!)

$\int c\sin(ax+b)\,dx$	$= -\frac{c}{a}\cos(ax+b) + K$	$\int 7\sin(8x+9)\,dx = -\frac{7}{8}\cos(8x+9) + K$
$\int sin^2x\,dx$ or $\int cos^2x\,dx$	Convert to $cos2x$ using $cos^2x - sin^2x = cos2x$ and $cos^2x + sin^2x = 1$. So $sin^2x = \frac{1}{2}(1-cos2x)$ or $cos^2x = \frac{1}{2}(1+cos2x)$	e.g. $\int sin^2x\,dx = \frac{1}{2}\int(1-cos2x)\,dx$ $= \frac{x}{2} - \frac{1}{4}sin2x + K.$ (note: this comes up on almost every paper!)
e.g. $\int cos^5x\,dx$ or any odd power of cos or sin.	Convert using $cos^2x = 1 - sin^2x$ $cos^5x = (1-sin^2x)^2cosx$	Expand to get $\int(1-2sin^2x+sin^4x)\cos x\,dx$ $= sinx - \frac{2}{3}sin^3x + \frac{1}{5}sin^5x + K$
e.g. $\int sin^3xcos^2xdx$ or any multiple powers of sin and cos.	Convert using $cos^2x + sin^2x = 1$ to give a single cos or sin.	$e.g. = \int sinx(1-cos^2x)cos^2xdx$ $= \int sinxcos^2x - sinxcos^4x\,dx$ $= -\frac{cos^3x}{3} + \frac{cos^5x}{5} + K$
e.g. $\int tan^5x\,dx$ or any power of $tan\ x$	Use $tan^2x = sec^2x - 1$	e.g. $= \int tan^3x(sec^2x-1)dx$ $= \int sec^2xtan^3x - \int tanx(sec^2-1)dx$ $= \frac{1}{4}tan^4x - \frac{1}{2}tan^2x + \ln\sec x + K$

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