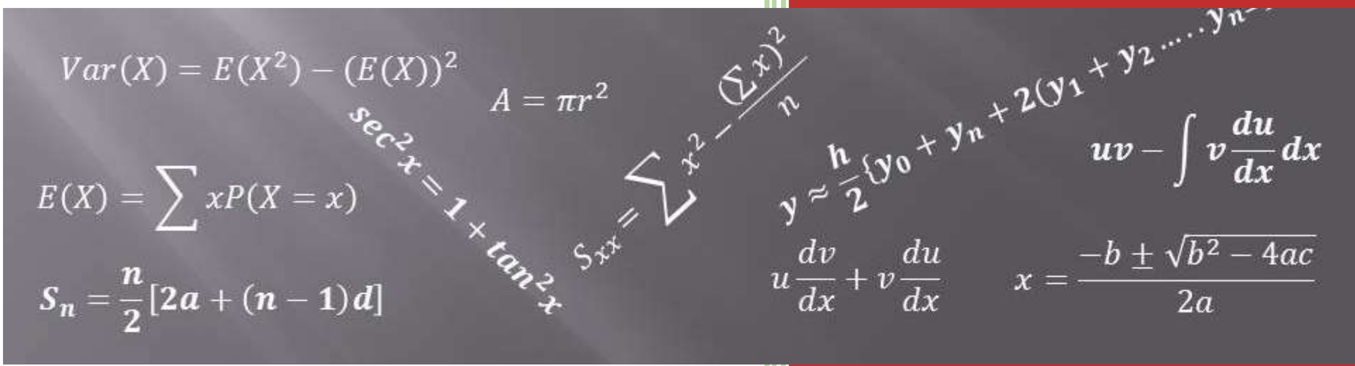
MathsGeeks

GCSE Higher Level Harder Questions Pack 1 – Solutions



Question No.	Mark Scored	Mark
1		5
2		4
3		8
4		6
5		4
6		4
7		5
8		7
9		7
10		6
11		4
12		3
13		4
14		6
15		3
16		4
17		5
18		7
19		5
20		3
TOTAL		**100**

Calculators Allowed

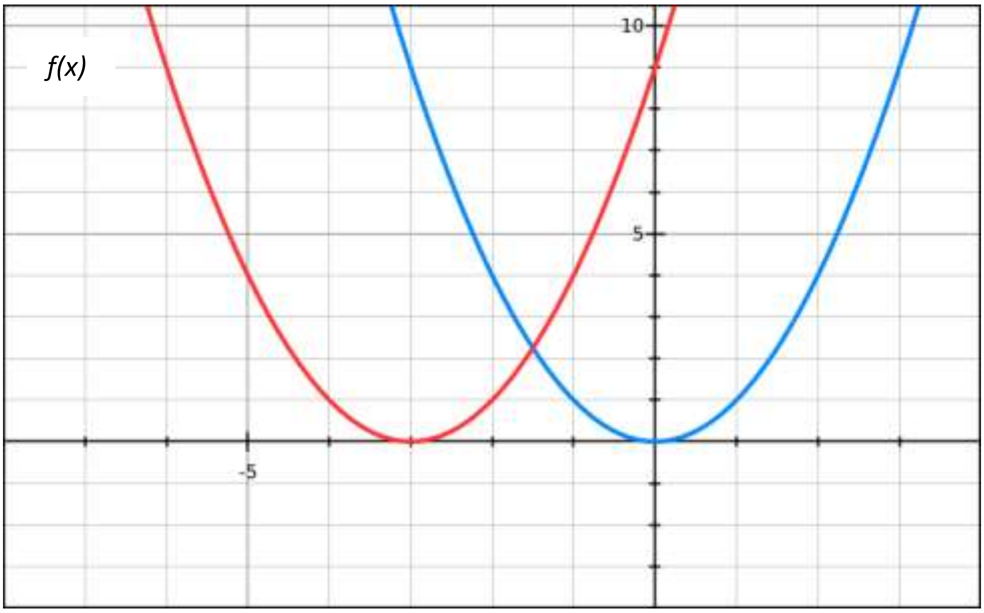
Time Allowed: 2hrs

Give all answer to 3 significant figures unless otherwise stated

September 2013

MathsGeeks

Question 1.

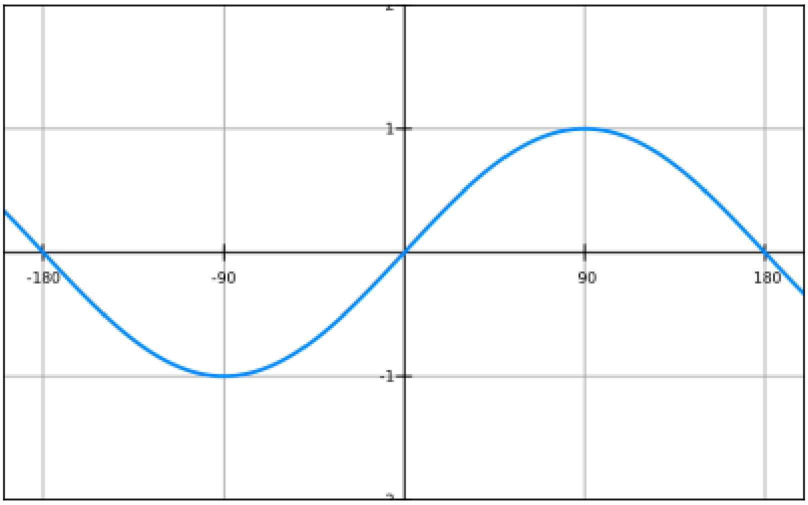


The curve with equation $y = f(x)$ (shown in red) is translated such that the point (-3,0) is mapped onto the point (0,0) (shown in blue).

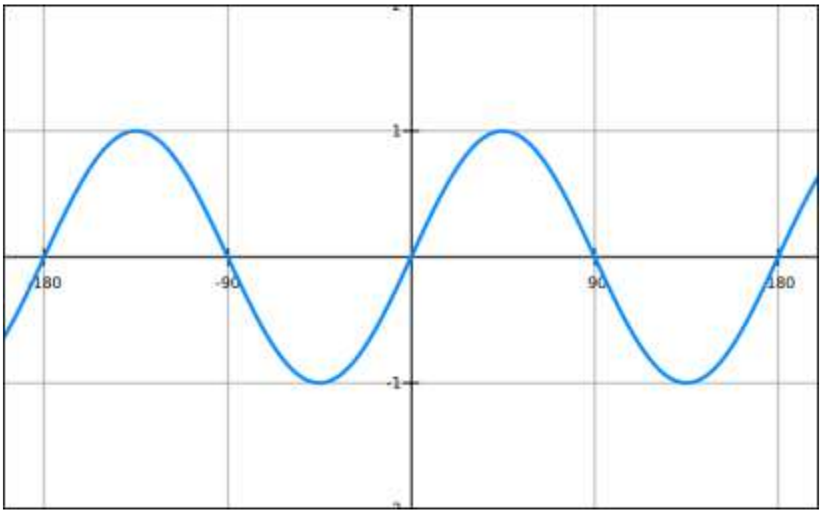
a) Find the equation of the translated curve.

The curve is shifted 3 positions to the left which it means it is shifted to $y = f(x-3)$ as x is counterintuitive. **(2)**

b) The graph shows y=sin(x) for values of x between -180 and 180°.

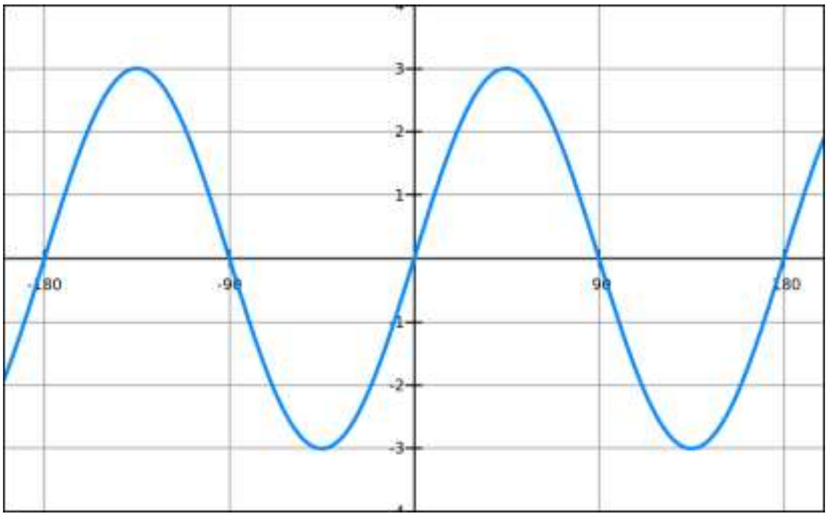


(i) Sketch the graph y=sin(2x) between –180° and 180°



(2)

(ii) Sketch the graph y=3sin(2x) between –180° and 180°



(1)

Question 1: TOTAL: /5

………

Question 2

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

a) Find f when $v = 4\frac{2}{3}$ and $u = 2\frac{1}{3}$

$v = 4\frac{2}{3} = \frac{14}{3}$ and $u = 2\frac{1}{3} = \frac{7}{3}$

Substitute these into the equation

$\frac{1}{14/3} = \frac{1}{f} - \frac{1}{7/3}$ re-arranging for $\frac{1}{f} = \frac{3}{14} + \frac{3}{7} = \frac{3+6}{14} = \frac{9}{14}$ and therefore $f = \frac{14}{9} = 1\frac{5}{9}$

(2)

b) Rearrange $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ to make f the subject of the formula.

Moving u to other side $\qquad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Multiple by vuf to get rid of the denominator $\qquad uf + vf = vu$

Factorise left hand side $\qquad f(u + v) = vu$

Divide by $u + v$ $\qquad f = \frac{vu}{u+v}$

(2)

Question 2: TOTAL: /4

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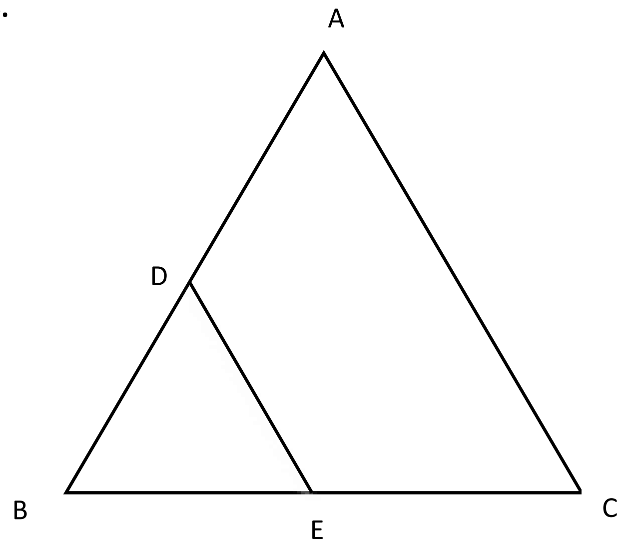
Question 3.



Not Drawn to Scale

Triangle BAC is an equilateral triangle.

D lies on AB such that AD=DB.

E lies on BC such that BE=EC.

a) Prove that triangle BDE and triangle BAC are similar.

To prove that two triangles are similar you must prove that two of the angles are equal (clearly this means the third angle is also equal). For different types of questions you can also prove that two of the sides are in the same ratio and one angle is the same, or all three sides are in the same ratio. Be sure to write a justification next to each statement.

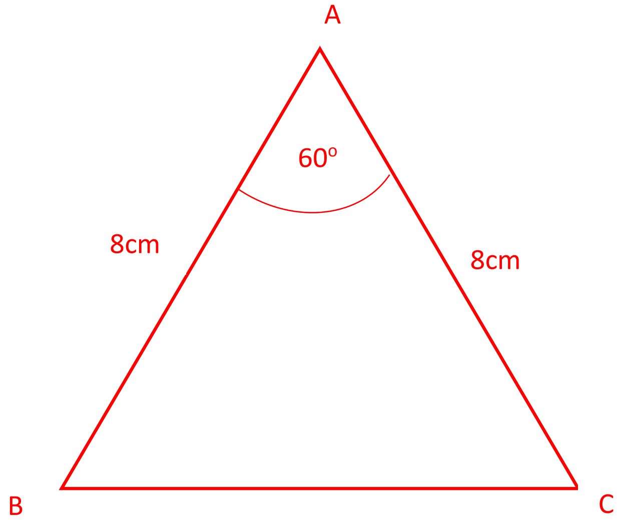
DBE = ABC - Same Angle

BDE = BAC = 60^o – As they are both equilateral triangles.

Therefore triangle BDE and triangle BAC are similar.

(3)

b) If BC=8cm find the area of triangle BAC.



Use the formula on the front page for triangles that are NOT right angles.

$Area = \frac{1}{2}ab\sin C$ where C is the angle BETWEEN length a and b.

Therefore $Area = \frac{1}{2}(8)(8)\sin 60 = 16\sqrt{3} = 27.7$ (3.s.f) cm^2

(3)

b) What is the area of BDE?

The area of BDE is $\frac{1}{4}$ of the area of BAC = $= 4\sqrt{3} = 6.93$ (3.s.f) cm^2

(2)

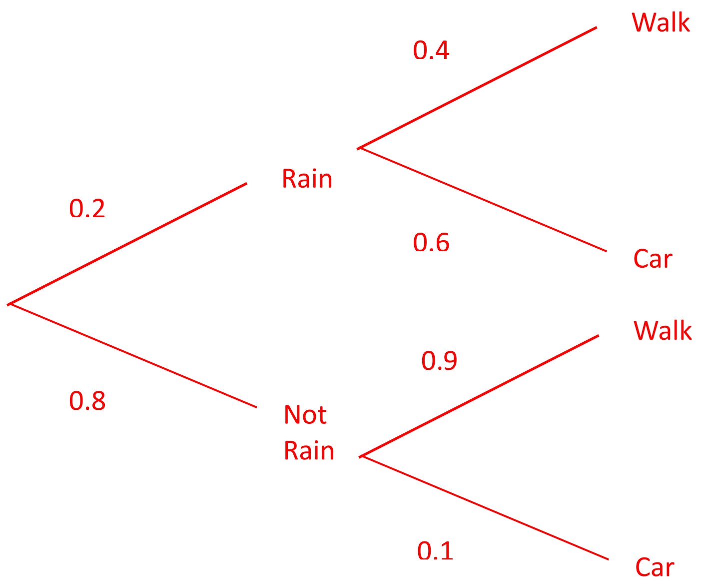
Question 3: TOTAL: /8

Question 4.

Sam either goes to school by car or he walks. If it rains the probability that he goes to school by car is 0.6 and if it doesn't rain the probability he goes by car is 0.1.

On a particular Tuesday the probability that it will rain is 0.2.

a) Draw a tree diagram to represent this information.



(4)

b) Work out the probability that he will walk to school.

The probability of walking is the top branch and the third branch added together which is

$$(0.2 \times 0.4) + (0.8 \times 0.9) = 0.8$$

(2)

Question 4: TOTAL: /6

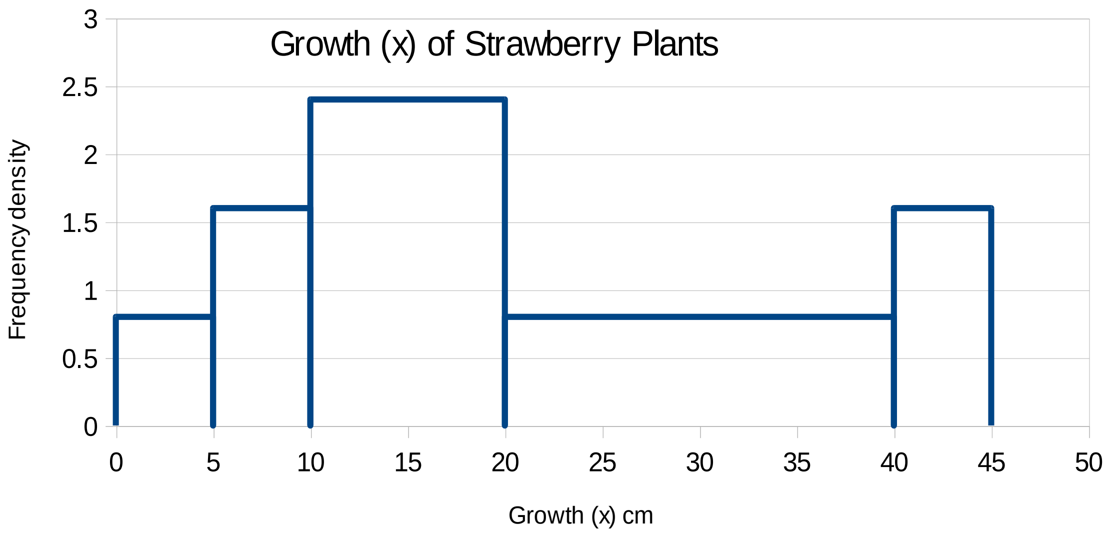
……

Question 5

A farmer plants strawberry plants and records their growth a year later. He records the following data:

Growth (x) cm	Frequency
$0 < x \leq 5$	4
$5 < x \leq 10$	8
$10 < x \leq 20$	24
$20 < x \leq 40$	16
$40 < x \leq 45$	8

Draw a frequency density histogram of this information.



Question 5: TOTAL: /4

Question 6.

X is inversely proportional to Y^2.

When X = 4, Y= 2.

a) Find a formula that relates X and Y.

As inversely proportional $X \propto \frac{1}{Y^2}$ which can be written as $X = \frac{k}{Y^2}$. Then put in the values of X and Y to find k.

$4 = \frac{k}{2^2}$ and $4 = \frac{k}{4}$ then $k = 16$ and the formula relating X and Y can be written as

$X = \frac{16}{Y^2}$. **(2)**

b) Find X when Y=12.

Put Y into the formula and find X.

$$X = \frac{16}{12^2} = \frac{16}{144} = \frac{1}{9}$$

(2)

Question 6: TOTAL: /4

....

Question 7.

a) Solve the inequality

$$8x + 3 < 5x + 21$$

Solve as if 'equal' to and then replace the sign. The only difference is when you divide by -1 then you must flip the sign i.e. $<$ becomes $>$.

$$8x + 3 = 5x + 21$$

Subtract 5x from both sides. $3x + 3 = 21$

Subtract 3 from each side. $3x = 18$ and $x = 6$.

Replace the sign so $x < 6$

(3)

b) x is whole number.

Write down the highest possible value that x can take.

$x < 6$ therefore $x = 5$. **(2)**

Question 7: TOTAL: /5

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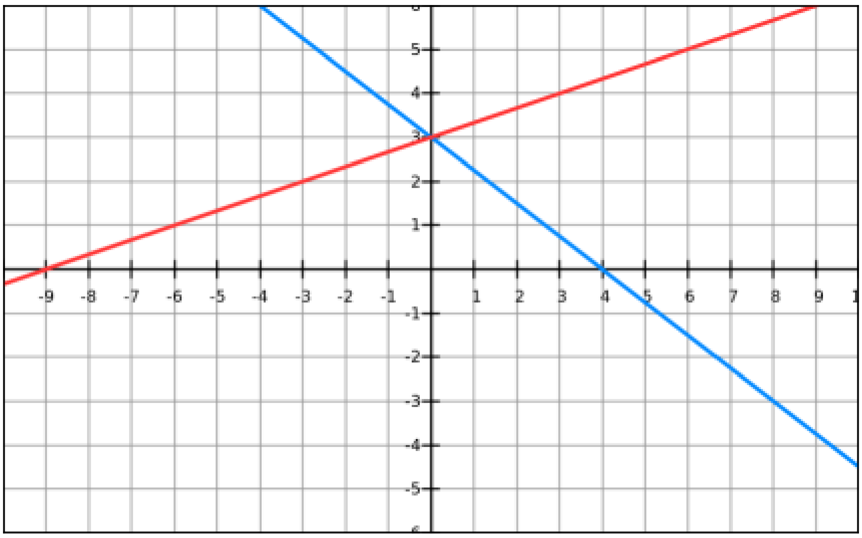
Question 8.

a) Plot, on the axes below, the lines:

$$3x + 4y = 12$$

$$y = \frac{x}{3} + 3$$



(4)

b) Using the graphs solve the simultaneous equations:

$$3x + 4y = 12$$

$$y = \frac{x}{3} + 3$$

This is the point where the two lines cross $x = 0, y = 3$

(1)

c) Find an equation of the straight line which is parallel to the line $y = \frac{x}{3} + 3$ and passes through the point (0,5).

If it is parallel to $y = \frac{x}{3} + 3$ it will have the same gradient but different intersect so $y = \frac{x}{3} + C$. Then substitute the point (0,5) into this to find C. $5 = 0 + C$ and therefore $y = \frac{x}{3} + 5$.

(2)

Question 8: TOTAL: /7

Question 9.

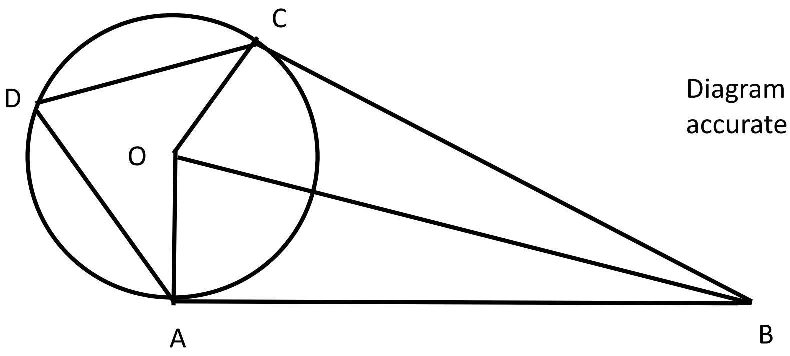


Diagram **NOT** accurately drawn

The diagram shows a circle with centre 0.
A, D and C are points on the circumference.
AB and CB are tangents to the circle.
The radius of the circle is 6cm.
The length of AB = 20cm.

a) Work out the size of the angle $\widehat{AOC}$.

First notice that triangle AOB is a right angle as AB is a tangent. Therefore we can find AOB using SOHCAHTOA.

tan $AOB = \frac{20}{6}$ and therefore $AOB = tan^{-1}\frac{20}{6} = 73.3$

Therefore AOC is twice AOB as symmetric. $AOC = 147^o\ (3.s.f)$.

(2)

b) Calculate the length of OB.

Using AOB is a right angled triangle use Pythagoras theorem.

i.e. $OB^2 = AB^2 + OA^2 = 6^2 + 20^2 = 436$ therefore $OB = \sqrt{436} = 20.9\ cm\ (3.s.f)$.

(2)

c) (i) Work out the size of the angle $A\widehat{D}C$

The angle at the circumference is half the angle at the centre. So $ADC = \frac{AOC}{2} = 73.3^o\ (3.s.f.)$ **(2)**

(ii) Give a reason for your answer.

Angle on the circumference of a circle is half the angle at the centre. **(1)**

Question 9: TOTAL: /7

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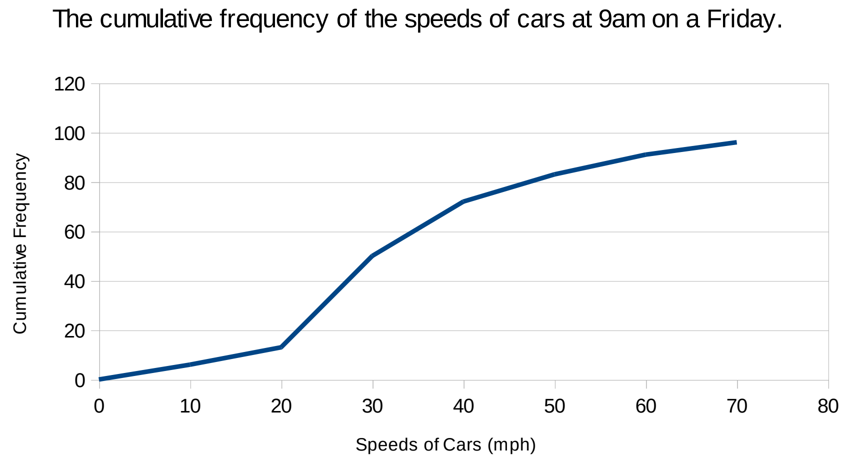
Question 10.

Radka did a survey of the speeds of 96 cars driving along a particular road from 9am on a Friday morning. The cumulative frequency table gives some information about the speeds of the 96 cars.

Speed (S mph)	Cumulative Frequency
$0 \leq S < 10$	6
$10 \leq S < 20$	13
$20 \leq S < 30$	50
$30 \leq S < 40$	72
$40 \leq S < 50$	83
$50 \leq S < 60$	91
$60 \leq S < 70$	96

a) On the grid, draw a cumulative frequency diagram.



(3)

a) Use your cumulative frequency diagram to estimate the median of the data.

The median value of the speeds is the 48th car which, reading from the graph, is approximately $30 \pm 2\ mph$. **(1)**

Radka then carried out a similar survey at 9am on the following Sunday morning and found that the median was 56 mph.

b) Compare the speeds of the cars on a Friday and Sunday and justify this difference.

The median speeds of the cars is far less on a Friday morning (30mph) than on a Sunday morning (56mph). This is because 9am on a Friday morning is 'rush hour' so there is much greater congestion and the average speed of the cars is much slower. **(2)**

Question 10: TOTAL: /6

Question 11.

Karim has 30 counters in a bag.

7 of the counters are black

10 of the counters are white

13 of the counters are red

Karim takes two counters from the bag.

Work out the probability that the counters will not be the same colour.

The easiest way is to work out the probability that the colours ARE the same and then take this from 1.

Probability of picking a black P(B) is $\frac{7}{30}$ if you have then removed one there are 6 left and only 29 in the bag so the probability of picking a second black is $\frac{6}{29}$.

Therefore $P(BB) = \frac{7}{30} \times \frac{6}{29} = \frac{7}{145}$.

Similarly $P(WW) = \frac{10}{30} \times \frac{9}{29} = \frac{3}{29}$

And $P(RR) = \frac{13}{30} \times \frac{12}{29} = \frac{26}{145}$

Therefore probability of same colour = $\frac{7}{145} + \frac{3}{29} + \frac{26}{145} = \frac{48}{145}$ and the probability of different colours $= 1 - \frac{48}{145} = \frac{97}{145}$.

(4)

Question 11: TOTAL: /4

Question 12.

Simplify fully

$$\frac{x^2 - x - 12}{2x^2 + 5x - 3}$$

First factorise the bottom and top of the fraction.

$$\frac{(x-4)(x+3)}{(2x-1)(x+3)}$$

Then cancel top and bottom therefore the fraction

$$\frac{(x-4)}{(2x-1)}$$

(3)

Question 12: TOTAL: /3

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Question 13.

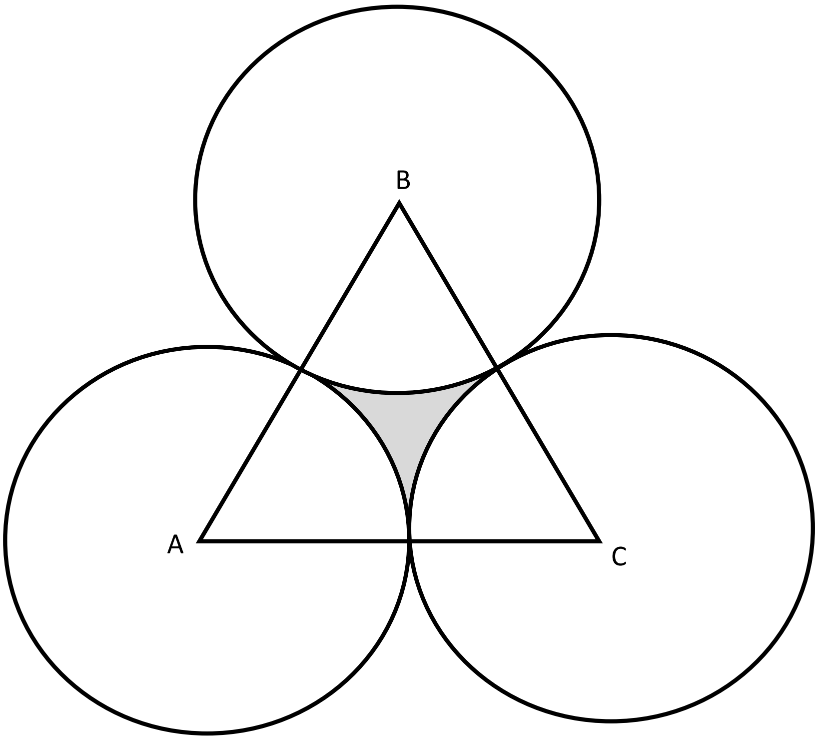

Diagram **NOT** accurately drawn

ABC is an equilateral triangle, where A, B and C are the centres of three circles of radius 4cm.
AB is length 8cm.
Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

The area of the shaded part is the area of the equilateral triangle minus the area of the three sectors.

As the triangle is equilateral all the angles are 60^0.

The area of a whole circle is = $\pi r^2 = \pi \times 4^2 = 50.265$. As there are 360^o in a circle then the area of the sector is given by = $\frac{60}{360} \times 50.625 = 8.3776$. The area of all three sectors is therefore $= 25.133$.

To find the area of the triangle use the formula on the front of the paper

$Area = \frac{1}{2} ab \sin C$ where a and b are lengths and C is the angle between the two lengths.

$Area = \frac{1}{2}(8)(8) \sin 60 = 27.713.$

The Area of the shaded is therefore $= 27.713 - 25.133 = 2.58\ (3.s.f)\ cm^2$.

(4)

Question 13: TOTAL: /4

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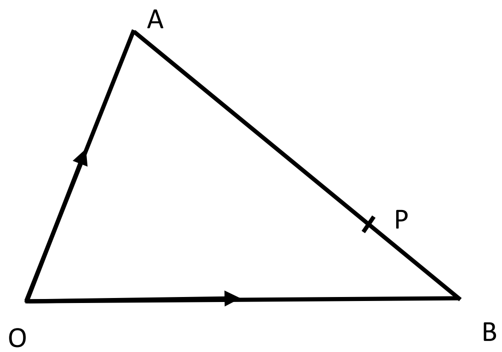
Question 14.



OAB is a triangle.

$$\overrightarrow{OA} = \boldsymbol{a}$$

$$\overrightarrow{OB} = \boldsymbol{2b}$$

a) Find the vector $\overrightarrow{AB}$ in terms of *a* and *b*.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\boldsymbol{a} + 2\boldsymbol{b} = 2\boldsymbol{b} - \boldsymbol{a}.$ **(2)**

P is the point on $\overrightarrow{AB}$ such that AP : PB = 4:1

b) Show that $\overrightarrow{OP}$ is

$$\overrightarrow{OP} = \frac{1}{5}(\boldsymbol{a} + 8\boldsymbol{b})$$

P is therefore $\frac{4}{5}$ of the way along AB so $AP = \frac{4}{5}(2\boldsymbol{b} - \boldsymbol{a})$.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \boldsymbol{a} + \frac{4}{5}(2\boldsymbol{b} - \boldsymbol{a}) = \frac{8\boldsymbol{b}}{5} + \frac{1}{5}\boldsymbol{a} = \frac{1}{5}(\boldsymbol{a} + 8\boldsymbol{b}).$

(4)

Question 14: TOTAL: /6

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Question 15.

Prove that $(5n + 2)^2 - (5n - 2)^2$ is a multiple of 8, for all positive integer values of n.

Start by writing out the brackets and then multiplying them out.

$(5n + 2)(5n + 2) - (5n - 2)(5n - 2) = (25n^2 + 10n + 10n + 4) - (25n^2 - 10n - 10n + 4).$

$= 20n + 20n = 40n = 8 \times 5n.$

Therefore this is always a multiple of 8. **(3)**

Question 15: TOTAL: /3

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Question 16.

366 students each study <u>one</u> of these science subjects at University.

The table shows the numbers of Males and Female students that studied each science subject.

	Science subject studied		
	Physics	Chemistry	Biology
Male	80	62	70
Female	42	50	62

In a survey a sample of 50 of the 366 students is taken.

a) Work out the number of female students studying Biology in the sample.

This is given by $\frac{Number\ of\ female\ students\ studying\ Biololgy}{Total\ number\ of\ students} \times Sample\ size.$

$= \frac{62}{366} \times 50 = 8.47$ which officially we round down to 8 students.

(2)

b) Work out the number of female students in the sample.

This is given by $\frac{Number\ of\ female\ students}{Total\ number\ of\ students} \times Sample\ size.$

$= \frac{42+50+62}{366} \times 50 = \frac{154}{366} \times 50 = 21.04$ which is rounded to 21 students.

(2)

Question 16: TOTAL: /4

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Question 17.

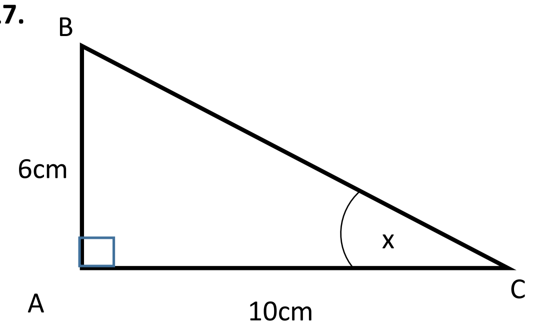

Diagram **NOT** accurately drawn

Triangle ABC is a right angled triangle.

(a) Calculate the size of the angle marked x.

Give your answer correct to 1 decimal place.

Using SOHCAHTOA we have the opposite and the adjacent so we use tan.

Therefore tan $x = \frac{6}{10}$ and therefore $x = tan^{-1}\left(\frac{3}{5}\right) = 30.964 = 31.0\ (1.d.p).$

x = ...° **(3)**

(b) Calculate the length BC.

Using Pythagoras Theorem $BC^2 = AB^2 + AC^2 = 6^2 + 10^2 = 136$

Therefore $BC = 11.66 = 11.7\ (1.d.p)$ cm

(2)

Question 17: TOTAL: /5

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Question 18.

a) Simplify

$$\frac{a \times a \times a}{a \times b}$$

One of the a's cancels top and bottom leaving $\frac{a \times a}{b} = \frac{a^2}{b}$.

(1)

(b) Expand

$$7(4y - 2)$$

Multiple out the brackets $28y - 14$.

(1)

(c) Expand

$$8x(x - 7)$$

Multiple out the brackets $8x^2 - 56x$.

(1)

(d) Expand and simplify

$$3(x - 8) + 4(x + 5)$$

First multiple out the brackets $3x - 24 + 4x + 20 = 7x - 4$

(2)

(e) Expand and simplify

$$(x-5)(x-8)$$

Multiple the 1st two terms, last two terms, middle two terms and outside two terms and then simplify.

$x^2 + 40 - 5x - 8x = x^2 - 13x + 40$.

(2)

Question 18: TOTAL: /7

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Question 19

The box plot gives information about the distribution of the weights of bags of toffees.

a) Rob says the heaviest bag weighs 37g.

He is wrong. Explain why.

The edge of the box (at 37g) is the upper quartile (Q_3)not the maximum value. The maximum value is the end of the line so here is 50g.

(1)

(b) Write down the median weight.

The median weight is where the middle line is which is 30g.

.... g **(1)**

(c) Work out the interquartile range of the weights.

The interquartile range is $Q_3 - Q_1 = 37 - 24 = 13 \pm 1g$.

.... g **(1)**

A factory produces 1024 bags of toffees per day.
(d) Work out the number of bags with a weight of 30g or less.

As 30g is the median that means that 50% of the bags are below this weight. Which means that $\frac{1024}{2} = 512$ bags.

(2)

Question 19: TOTAL: /5

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Question 20.

Prove that the recurring decimal $0.\dot{6}\dot{3}$ is $\frac{7}{11}$.

Let $n = 0.\dot{6}\dot{3} = 0.636363636363$ … …

If we then multiple this number by 100 we get $100n = 63.636363636363$ … ….

If we then carry out $100n - n = 63$ therefore $99n = 63$ and $n = \frac{63}{99} = \frac{7}{11}$.

(3)

Question 20: TOTAL: /5

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