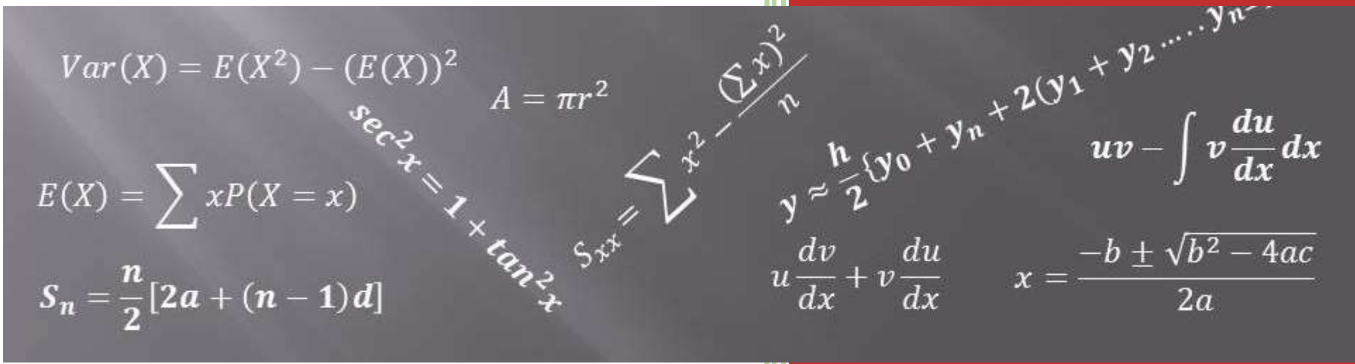
MathsGeeks

GCSE Higher Level Harder Questions Pack 2 - Solutions



Question No.	Mark Scored	Mark
1		8
2		4
3		6
4		4
5		4
6		5
7		4
8		5
9		7
10		5
11		5
12		4
13		4
14		3
15		4
16		5
17		4
18		7
19		7
20		5
TOTAL		**100**

Calculators Allowed

Time Allowed: 2hrs

Give all answer to 3 significant figures unless otherwise stated

September 2013

MathsGeeks

Question 1.

(a) Simplify

$$\frac{m^7 \times m^5}{m^2}$$

If you multiply indices you add them and if you divide you subtract so:

$$\frac{m^{12}}{m^2} = m^{10}.$$

(2)

(b) Simplify

$$(2x)^3 \times (3x)^2$$

$2x \times 2x \times 2x \times 3x \times 3x = 2 \times 2 \times 2 \times 3 \times 3 \times x^5 = 72x^5.$

(2)

(c) Simplify

$$5a^2bc^3 \times 6ab^4c^2$$

$$= 5 \times 6 \times a^2 \times a \times b \times b^4 \times c^3 \times c^2 = 30a^3b^5c^5$$

(2)

(d) Factorise completely

$$6x^2y^2z^3 + 18x^3y^2z^2$$

Find what the terms that they have in common and bring these out the front of the bracket.

$= 6x^2y^2z^2(z + 3x).$

(2)

Question 1: TOTAL: /8

………

Question 2.

Solve the equation

$$6x^2 + 9x - 13 = 0$$

Give each solution correct to 2 decimal places.

To solve a quadratic where they ask to 2 decimal places it clearly doesn't factorise and you will have to use the formula. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, where $a = 6, b = 9, c = -13$.

Therefore $x = \frac{-9 \pm \sqrt{81-4(6)(-13)}}{12}$

Therefore $x_1 = \frac{-9+\sqrt{393}}{12} = 0.90202 = 0.90\ (2.d.p)$, $x_2 = \frac{-9-\sqrt{393}}{12} = -2.40202 = -2.40\ (2.d.p)$

(4)

Question 2: TOTAL: /4

....

Question 3.

Julie invested £4500 for 3 years in a savings account.

She was paid 3% per annum compound interest.

a) How much did Julie have in his savings account after 3 years?

For compound interest each year she would receive 3% added.

So after 1 year it would be $4500 + \frac{3}{100} \times 4500 = 4635$

And so for the 2nd year it would be $4635 + \frac{3}{100} \times 4635 = 4774.05$ and so on for the 3rd year.

The easy way to do this is $4500 \times (1.03)^3 = 4917.2715$ = £4917.27 to the nearest penny.

£ .... **(3)**

Julie also invested £3000 for n years in another savings account.

She was paid 8% per annum compound interest.

At the end of the n years she had £4407.98 in the savings account.

(b) Work out the value of n.

Each year the amount will increase by 0.08 so the total amount will be 1.08, so we see how many times we multiple 3000 by 1.08 to get to 4407.98.

$= 3000 \times 1.08 = 3240.00$

$= 3240 \times 1.08 = 3499.20$

$= 3499.20 \times 1.08 = 3779.14$

$= 3779.14 \times 1.08 = 4081.47$

$= 4081.47 \times 1.08 = 4407.98$

and therefore n=5 as 5 years have past.

(3)

Question 3: TOTAL: /6

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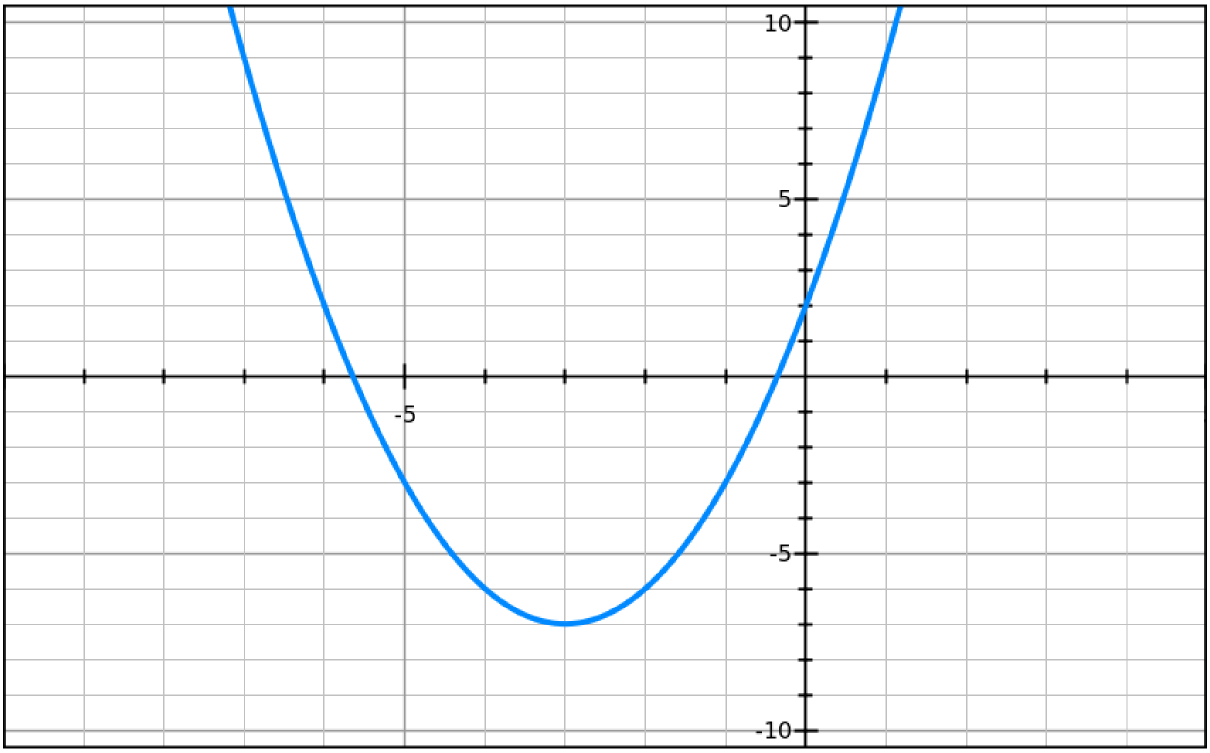
Question 4.

This is a sketch of the curve with the equation y = f(x).



The only minimum point of the curve is at $P(-3, -7)$.

(a) Write down the coordinates of the minimum point of the curve with the equation $y = f(x + 2)$

If $f(x)$ goes to $f(x + 2)$ this is a shift of two to the LEFT so the new minimum point is $(-5, -7)$.

(………… , …………) **(2)**

(b) Write down the coordinates of the minimum point of the curve with the equation

$y = f(x + 2) + 5$

As the +5 is outside the bracket it is a shift in y which moves the curve upwards 5 points. The new minimum point is therefore $(-5, -2)$.

(………… , …………) **(2)**

Question 4: TOTAL: /4

………

Question 5.

Simplify fully

$$\frac{x^2 + 9x + 20}{x^2 - 16}$$

First factorise the top and bottom of the equation. Notice that the bottom is the difference between two squares.

$$= \frac{(x + 4)(x + 5)}{(x - 4)(x + 4)}$$

And then the $(x + 4)$ cancels to give

$$= \frac{(x + 5)}{(x - 4)}$$

(4)

Question 5: TOTAL: /4

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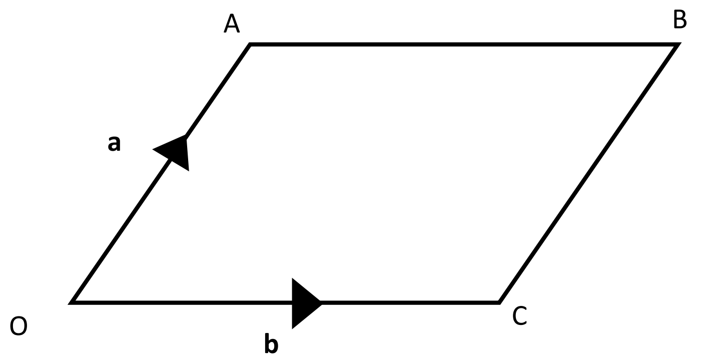
Question 6.

OABC is a parallelogram



OA = **a**, OC = **b**

(a) Find the vector $\overrightarrow{OB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ and $\overrightarrow{AB} = \overrightarrow{OC}$ therefore $\overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{b}$.

$\overrightarrow{OB} =$ .... **(2)**

P is the point on $\overrightarrow{OB}$ so that $OP : PB = 3 : 2$

(b) Find the vector $\overrightarrow{AP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Give your answer in its simplest form.

$$\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP} = -\boldsymbol{a} + \frac{3}{5}(\boldsymbol{a} + \boldsymbol{b}) = -\frac{2}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b} = \frac{1}{5}(2\boldsymbol{a} + 3\boldsymbol{b})$$

(Note: if you are writing as opposed to typing you should really underline each vector so vectors ***a*** would be written as ***<u>a</u>***.

$\overrightarrow{AP}$ = ... **(3)**

Question 6: TOTAL: **/5**

Question 7.

Work out

$$(\sqrt{5}+3)(\sqrt{5}-3)$$

Give your answer in the simplest form.

Multiple 1st two terms, last two terms, middle two terms and outside terms to get –

$\sqrt{5\sqrt{5}} - 9 + 3\sqrt{5} - 3\sqrt{5} = 5 - 9 = -4$

.... (4)

Question 7: TOTAL: /4

....

Question 8.

Solve the simultaneous equations

$$x - 3y = 10$$

$$3y + 2x = 8$$

To solve simultaneous equations we need to 'get rid' of either x or y. By inspection we can see that by writing equation two in a different order we have

$$x - 3y = 10$$

$$2x + 3y = 8$$

If we then add these two equations then the y values are removed so

$x + 2x = 10 + 8$ and $3x = 18$ and so $x = 6$.

We then substitute this back into the first equation i.e. $6 - 3y = 10$ and rearrange so $6 - 10 = 3y$.

$-4 = 3y$ and $y = -\frac{4}{3}$. The solution is therefore $x = 6, y = -\frac{4}{3}$ $(-1\frac{1}{3})$.

Question 8: TOTAL: /5

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Question 9.

Elwin has 12 pens in a box.

5 of the pens are blue.

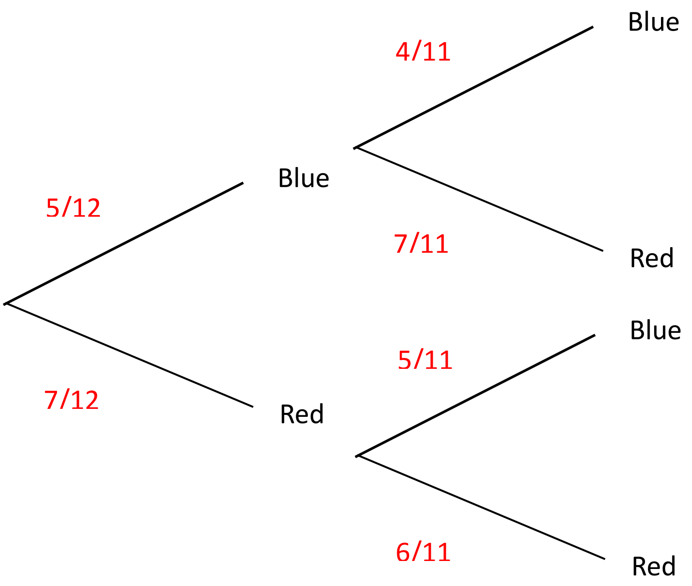
7 of the pens are red.

Elwin takes at random a pen from the box and writes down its colour.

Elwin DOES NOT put the pen back in the box.

Then Elwin takes at random a second pen from the box, and writes down its colour.

a) Complete the probability tree diagram.



(4)

b) Work out the probability that Elwin takes exactly one pen of each colour from the box.

To get exactly one of each colour she can either pick a Blue then a Red or a Red then a Blue

If you follow the branch of Blue then Red you get $= \frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$

And the branch of Red then Blue $= \frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$

Therefore the probability of exactly one colour is $\frac{35}{132} + \frac{35}{132} = \frac{70}{132} = \frac{35}{66}$.

.... **(3)**

Question 9: TOTAL: /7

Question 10.

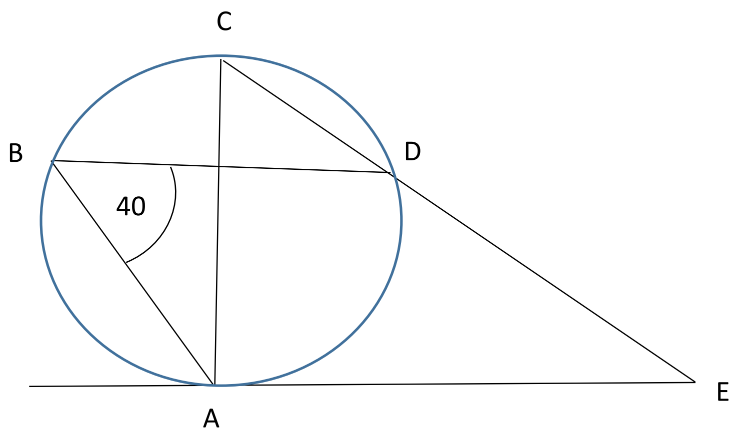


Diagram **NOT** accurately drawn

A, B, C, and D lie on a circle.

AE is a tangent to the circle.

AC is the diameter of the circle.

Angle ABD = 40°.

(a) Find the size of angle ACE.

Give a reason for your answer.

The angle $ACE = ABD = 40^o$

Reason: angles subtended by the same chord of a circle (the chord in this case is AD) OR angles on the same segment.

..............................° **(2)**

(b) Find the size of angle CEA.

Give a reason for your answer.

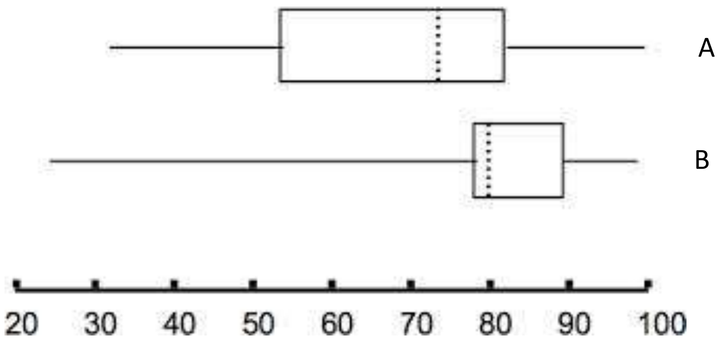
As AE is a tangent to the circle angle CAE is 90° and using our answer from part a) we find that angle CEA is 50° as the angles in a triangle add up to 180°.

..............................° **(3)**

Question 10: TOTAL: /5

Question 11.

The box plots show the distribution of weights (kg) of two different groups (A and B) of athletes.



a) What is the largest weight of an athlete in group A?

The largest weight is where the range bar ends which is the line through the middle, so the largest weight of an athlete in group A is 100kg.

....................................kg **(1)**

b) What is the interquartile range of the data for group B.

The interquartile range is the upper quartile minus the lower quartile which is also the size of the box so it is $88 - 78 = 10 \pm 2$kg.

....................................kg **(2)**

c) Compare the distributions of the weight of the two groups of athletes A and B.

There are many possible answers but there are two marks so ensure you make two distinct points.

1. The Range of B ($99 - 24$ =75kg) is greater than the Range of A $(100 - 32 = 68kg)$.

2. The interquartile range (IR) of B (10kg) is much smaller than the interquartile range (IR) of A $(83 - 53 = 30kg)$.

3. The median weight of the athletes in group B (80kg)is greater than the median weight of the athletes in group A (74kg).

(2)

Question 11: TOTAL: /5

Question 12.

Make x the subject of the formula

$$6(x + y) = 3 - 8x$$

Give your answer in its simplest form.

Using BODMAS we start by multiplying out the brackets to give $6x + 6y = 3 - 8x$

We then move all the x's to one side (choose the largest side) so $6x + 8x + 6y = 3$ and

Simplifying $14x + 6y = 3$

Move everything else to the other side $14x = 3 - 6y$

Then divide by 14 to get x by itself $x = \frac{3-6y}{14}$

x = ... (4)

Question 12: TOTAL: /4

....

Question 13.

a) Write in standard index form 5, 670, 000

For standard index form you must have only one number before the decimal point and then multiple by a multiple of 10. e.g. 5.67×10^6.

.... **(1)**

b) Write in standard index form 0.000167

$= 1.67 \times 10^{-4}$.

.... **(1)**

c) Work out $\frac{3.12\times10^2+5.6\times10^3}{1.34\times10^4}$ writing your answer in standard form

It is easier to see this calculation in normal form so converting we have

$$\frac{312 + 5600}{13400} = \frac{5912}{13400} = 0.4411940 = 4.41 \times 10^{-1}$$

.... **(2)**

Question 13: TOTAL: /4

....

Question 14.

(a) Write down the value of 6^0.

Any number to the power of 0 = 1 so $6^0 = 1$

………………………………………. (1)

(b) Write down the value of

$$\left(\frac{4}{3}\right)^{-1}$$

Any fraction to the power of -1 means turn it upside down so

$$\left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$$

………………………………………. (2)

Question 14: TOTAL: /3

……

Question 15.

k is an integer such that $-2 \leq k < 2$

(a) List all the possible values of k.

This means that k is greater than or equal to -2 and less than 2 so it can take the values

-2, -1, 0 and 1.

…………………………………………………. **(2)**

(b) Solve the inequality

$$7y \geq -y + 16$$

Solve as if it is 'equal to' instead (as long as you don't divide by a minus number) and then just put the sign back in. Therefore

$7y = -y + 16$ and $7y + y = 16$ so $8y = 16$ therefore $y = 2$. Substituting the sign back in

$$y \geq 2.$$

…………………………………………………. **(2)**

Question 15: TOTAL: /4

……

Question 16.

$$y = \frac{a^2}{b}$$

a = 7.35 correct to 2 decimal places.

b = 6.789 correct to 3 decimal places.

By considering bounds, work out the maximum possible value of y to 3 decimal places.

You must show all your working and give a reason for your final answer.

If you divide by a smaller number you get a bigger number so

$$y_{max} = \frac{{a_{max}}^2}{b_{min}}$$

The maximum value of $a = 7.355$ and the minimum value of $b = 6.7885$.

Therefore

$$y_{max} = \frac{{a_{max}}^2}{b_{min}} = \frac{7.355^2}{6.7885} = 7.969\ (3.d.p)$$

y = ... **(5)**

Question 16: TOTAL: /5

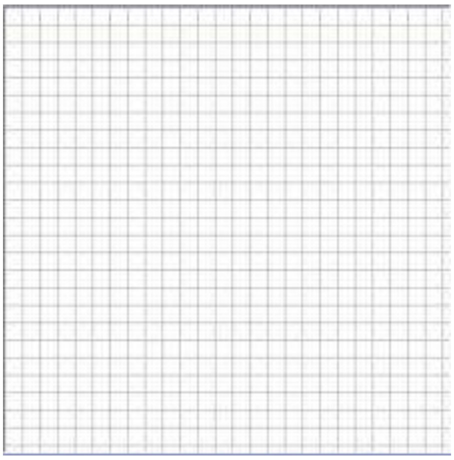
....

Question 17.

The table gives some information about the time, in minutes, it takes 100 year 11 students to walk to school.

Time (t) minutes	Frequency
$0 < t \leq 5$	5
$5 < t \leq 10$	10
$10 < t \leq 20$	25
$20 < t \leq 30$	45
$30 < t \leq 45$	9
$45 < t \leq 60$	6

Use the information to construct a frequency density histogram.



(4)

Question 17: TOTAL: /4

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Question 18.

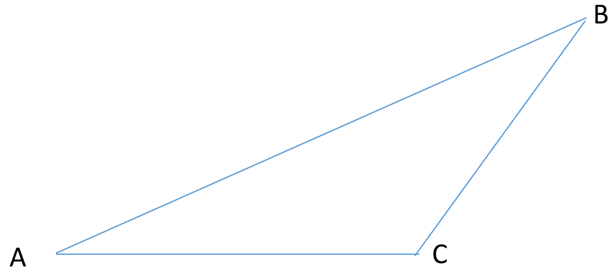

Diagram **NOT** accurately drawn

In triangle ABC,

AC = 5.5 cm.

BC = 4.4 cm.

Angle ACB = 109°.

a) Calculate the area of triangle ABC.

Give your answer correct to 3 significant figures.

The area of a scalene triangle is given by the formula on the front of the paper.

$Area\ = \frac{1}{2}ab\sin C$ where the angle C is the angle between lengths a and b.

$Area\ = \frac{1}{2}(5.5)(4.4)\sin 109 = 11.4\ (3.s.f)$

.... cm^2 **(3)**

b) Calculate the length of AB.

Give your answer correct to 3 significant figures.

To find a length in a scalene triangle we have to use the formulas on the front of the exam paper. If you have more lengths than angles you should use the *cos* formula i.e.

$a^2\ = b^2 + c^2 - 2bc\cos A$ where length a is opposite angle A. Therefore

$a^2\ = 5.5^2 + 4.4^2 - 2(5.5)(4.4)\cos 109 = 8.085 = 8.09\ (3.s.f)\ \ cm.$

.... cm **(3)**

c) Calculate the perimeter of triangle ABC

Give your answer correct to 3 significant figures.

The perimeter is therefore just $AB + BC + CA = 5.5 + 4.4 + 8.09 = 18.0\ (3.s.f)$ cm.

.... cm **(1)**

Question 18: TOTAL: /7

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Question 19.

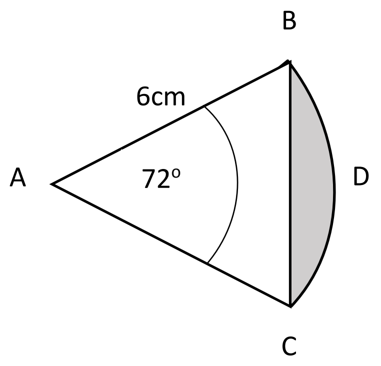

Diagram **NOT** accurately drawn

BAC is a sector of a circle, radius 6 cm.

The angle BAC is 60°.

a) Find the length of BC.

The easiest way to do this is to use the formula for lengths in a scalene triangle. As we only have one length we will use the *cos* formula.

$a^2 = b^2 + c^2 - 2bc\cos A = 6^2 + 6^2 - 2(6)(6)\cos 72 = 49.75$

$a = BC = 7.05\ (3.s.f)cm.$

.... cm **(3)**

b) Work out the area of the shaded region BDC.

The area of the shaded shape is the area of the sector minus the area of triangle.

The area of the sector is going to be $\frac{72}{360} \times \pi \times 6 \times 6 = 22.6\ cm^2$.

The area of the triangle is given by the formula on the front of the exam paper

$$Area = \frac{1}{2} ab \sin C = \frac{1}{2}(6)(6) \sin 72 = 17.119\ cm^2.$$

Therefore the area of the shaded shape is $22.6 - 17.119 = 5.48\ (3.s.f) cm^2$.

Area = ...cm^2 **(4)**

Question 19: TOTAL: /7

....

Question 20.

a is directly proportional to *b*.

When *a* = 1000, *b* = 25

(a) Find a formula for *a* in terms of *b*.

$a \propto b$
$a = kb$
$1000 = k \times 25$ and $k = 40$

Therefore $a = 40b$

a = ... (3)

(b) Calculate the value of *a* when *b* = 350

Simply substitute b into the formula you derived above. So

$$a = 40 \times 350 = 14000$$

a = ... (2)

Question 20: TOTAL: /5

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