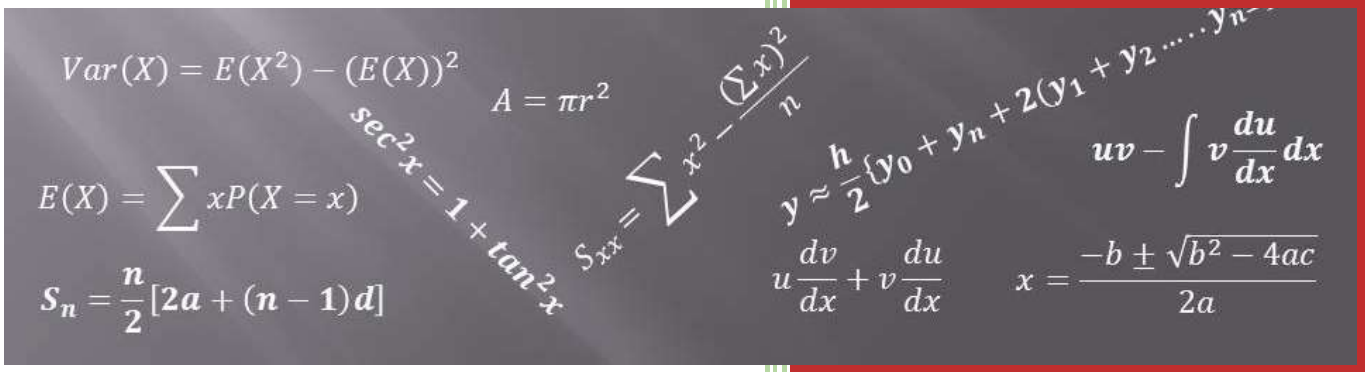
MathsGeeks

GCSE Higher Level Harder Questions Pack 3 - Questions



Question No.	Mark Scored	Mark
1		2
2		6
3		5
4		5
5		4
6		8
7		6
8		7
9		4
10		8
11		5
12		7
13		3
14		3
15		4
16		2
17		4
18		6
19		7
20		4
TOTAL		**100**

Calculators Allowed

Time Allowed: 2hrs

Give all answer to 3 significant figures unless otherwise stated

September 2013

MathsGeeks

Copyright © Elite Learning Limited

Question 1.

The table below gives some information about some students in a school.

Year group	**Boys**	**Girls**	**Total**
Year 10	52	65	117
Year 11	61	67	128
Total	113	132	345

Chang is going to carry out a survey of these students.

He uses a sample of 50 students, stratified by year group and gender.

Work out the number of Year 11 girls that should be in his sample.

The number of year 11 girls is 67 so this is $\frac{67}{345} \times 50 = 9.71$ which rounds to 10 girls.

…… (2)

Question 1: TOTAL: /2

……

Question 2.

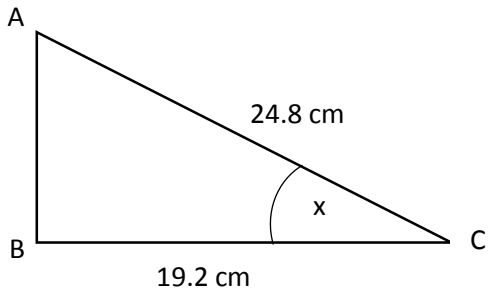


Diagram **NOT** accurately drawn

ABC is a right-angled triangle.
AC = 24.8 cm.
BC = 19.2 cm.

a) Calculate the size of the angle marked *x*.
Give your answer correct to 3 significant figures

Copyright©2013 Elite Learning Ltd

As this a right angled triangle we can use SOHCAHTOA.

We have the hypotenuse and the adjacent so we will use cos.

Therefore $\cos x = \frac{a}{h} = \frac{19.2}{24.8} = \frac{24}{31}$

And solving for x we get $x = cos^{-1}\frac{24}{31} = 39.268 = 39.3^o\ (3.s.f.)$

……° **(3)**

b) Calculate the area of triangle ABC.

The area of a right angled triangle is

$Area = \frac{1}{2} \times base \times height = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 19.2 \times AB$

To find AB we can use Pythagoras theorem so $AC^2 = AB^2 + BC^2$ and rearrange for AB therefore

$AB^2 = AC^2 - BC^2 = 24.8^2 - 19.2^2 = 246.4$ and therefore $AB = \sqrt{246.4} = 15.697 = 15.7\ (3.s.f)\ cm^2$.

Therefore the area of the triangle is $= \frac{1}{2} \times 19.2 \times 15.697 = 151\ (3.s.f)$.

……..cm² **(3)**

Question 2: TOTAL: /6

Copyright©2013 Elite Learning Ltd

Question 3.

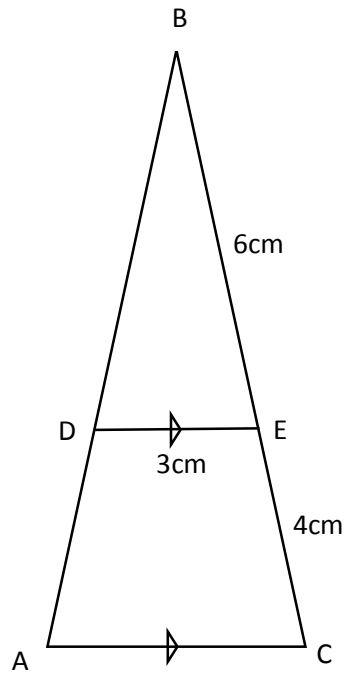


Diagram **NOT** accurately drawn

ABC is an isosceles triangle.
DE is parallel to AC.

a) Prove that triangle DBE is similar to triangle ABC.

To prove that two triangles are *similar* you must prove that at least one angle is the same and that two lengths are in the same ratio OR that two angles are the same (this implies the third angle is also the same).

Angle ABC = Angle DBE as it is the same angle.

Angle CAB = Angle EDB corresponding angles.

Angle BCA = Angle BED corresponding angles. **(3)**

b) Find the length AC.

As the triangles are similar and DE (3) is half of BE (6) then AC is half of BC (6+4).

And so AC = 5cm

…….. cm **(2)**

Question 3: TOTAL: /5

Copyright©2013 Elite Learning Ltd

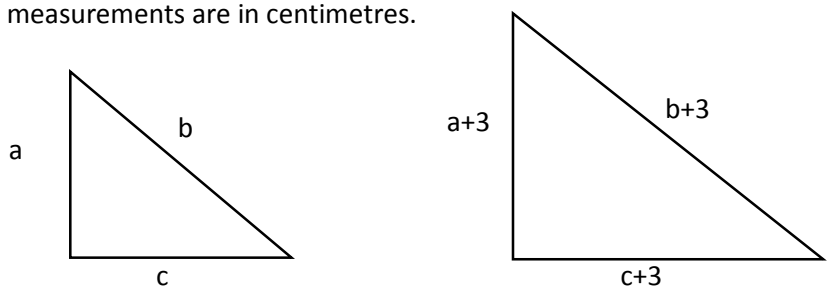
Question 4
Howard thinks that $(x + 3)^2 = x^2 + 9$ for all values of *x*.

a) Show that Howard is wrong.
All you need to do is prove ANY inconsistency.
Start by expanding out the left hand side.

$(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9$
$= x^2 + 6x + 9 \neq x^2 + 9$ therefore Howard is wrong.

(2)

Here are two right-angled triangles.
All the measurements are in centimetres.



b) Show that $2a + 2c = 2b - 3$

From the triangle on the left using Pythagoras' theorem we find that

$a^2 + c^2 = b^2$ and from the triangle on the right we have

$(a + 3)^2 + (c + 3)^2 = (b + 3)^2$ expanding this out we get

$a^2 + 6a + 9 + c^2 + 6c + 9 = b^2 + 6b + 9$

But from the first triangle we see that $a^2 + c^2 = b^2$so this becomes

$b^2 + 6a + 9 + 6c + 9 = b^2 + 6b + 9$

The b^2 cancel on both sides, as do one of the 9s to leave.

$6a + 6c + 9 = 6b$

If we then divide by 3 we get $2a + 2c + 3 = 2b$

Then we take the 3 to the other side $2a + 2c = 2b - 3$

(3)

Question 4: TOTAL: /5

Copyright©2013 Elite Learning Ltd

Question 5
Expand and simplify

$$(2 + \sqrt{3})(3 + \sqrt{12})$$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

Multiply out each term as you would a quadratic (first two terms, last two terms, middle terms and edge terms).

$$(2 + \sqrt{3})(3 + \sqrt{12}) = 6 + 3\sqrt{3} + 2\sqrt{12} + \sqrt{3}\sqrt{12}$$

(Note by convention we write the number in front of the surd).

But $\sqrt{3\sqrt{12}} = \sqrt{36} = 6$ so the whole expression becomes.

$$= 12 + 3\sqrt{3} + 2\sqrt{12}$$

Notice that $\sqrt{12}$ can be written as $\sqrt{4} \times 3 = 2\sqrt{3}$ therefore $2\sqrt{12} = 4\sqrt{3}$.

The expression is therefore

$$= 12 + 3\sqrt{3} + 4\sqrt{3} = 12 + 7\sqrt{3}$$

..... **(4)**
Question 5: TOTAL: **/4**

Copyright©2013 Elite Learning Ltd

Question 6.
Here is a shape ABCDE

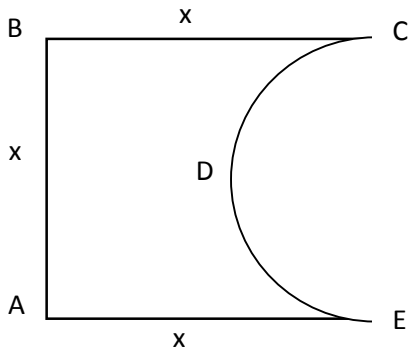


Diagram **NOT** accurately drawn

AE, A*B* and *BC* are three sides of a square.
AB = *x* cm.
EDC is a semicircle with diameter *CE*.

a) Show that the perimeter, *P* cm, of the shape *ABCDE* is given by the formula

$$P = 3x + \frac{\pi x}{2}$$

The perimeter of the three sides of the square is $3x$. The radius of the semi-circle is clearly $\frac{x}{2}$.

The circumference of the whole circle is given by $2 \times \pi \times r = 2 \times \pi \times \frac{x}{2} = \pi x$ but CDE is half the circle so is length $\frac{\pi x}{2}$.

The total perimeter is therefore $P = 3x + \frac{\pi x}{2}$.

(3)

b) Rearrange this formula to make x the subject.

Start by factorising the right hand side so

$P = x(3 + \frac{\pi}{2})$

Then divide through by the bracket so $\frac{P}{3+\frac{\pi}{2}} = x$.

You can multiply up by 2 to give $\frac{2P}{6+\pi} = x$.

..... **(3)**

Copyright©2013 Elite Learning Ltd

The area, A cm^2, of this shape is given by $A = kx^2$ where k is a constant.

c) Find the exact value of k.
Give your answer in its simplest form.

The area A is the area of the square minus the area of the semi-circle.

The area of the square is just x^2.

The area of a circle is given by πr^2 but in this case $r = \frac{x}{2}$ and the area = $\pi(\frac{x}{2})^2 = \frac{\pi x^2}{4}$

So the area of the semi-circle is a half of this value i.e $= \frac{\pi x^2}{8}$

The total area of the shape is $A = x^2 - \frac{\pi x^2}{8} = x^2(1 - \frac{\pi}{8})$ and therefore $k = 1 - \frac{\pi}{8}$.

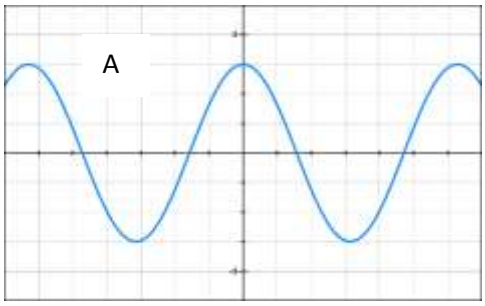
..... **(2)**

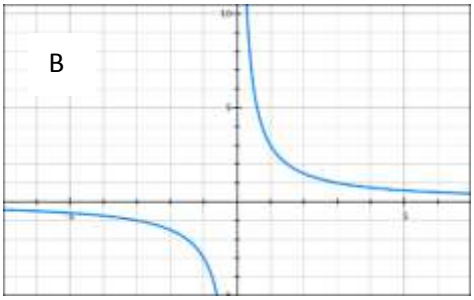
Question 6: TOTAL **/8**

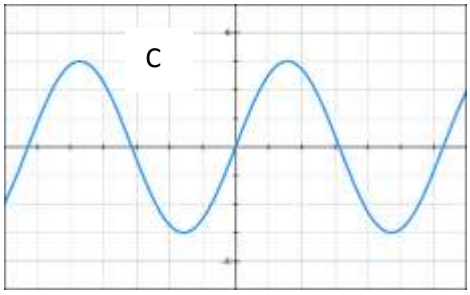
.....

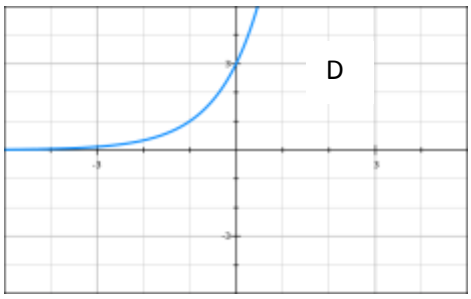
Question 7

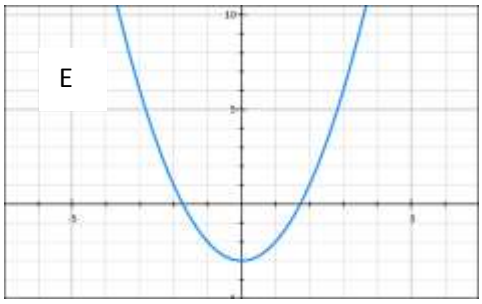
Each equation in the table represents one of the graphs A to F.
Write the letter of each graph in the correct place in the table.

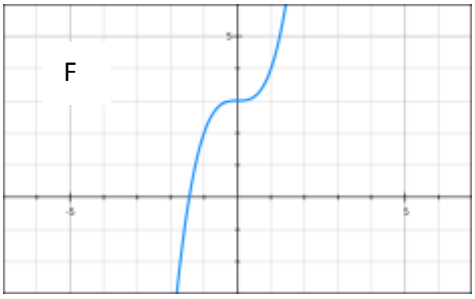












Copyright©2013 Elite Learning Ltd

Equation	Graph
$y = x^2 - 3$	E
$y = \frac{3}{x}$	B
$y = 3\sin x$	C
$y = 3\cos x$	A
$y = 3 \times 3^x$	D
$y = x^3 + 3$	F

Question 7: TOTAL: /6

.....

Question 8.

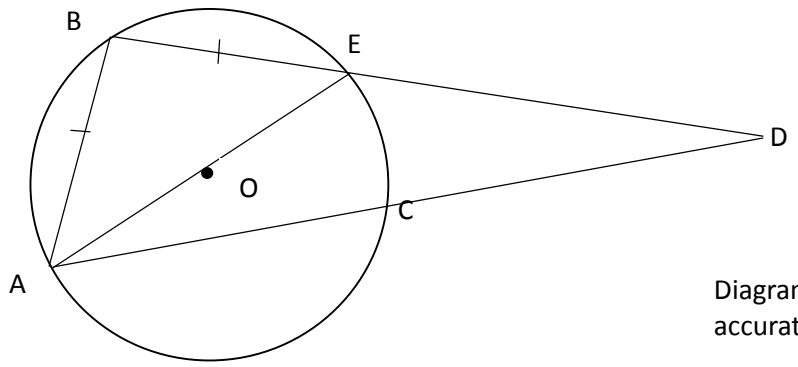


Diagram **NOT** accurately drawn

Points A, B and E lie on a circle with centre O.

Triangle ABE is an isosceles triangle.

AE is the diameter of the circle.

AE = 10cm.

AD = 30cm.

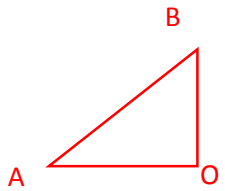
a) Calculate angle BAE.

As the angle ABE is subtended by the diameter AE it has to be a right angle (90°). We are also told that ABE is isosceles so BAE=BEA. As angles in a triangle add up to 180° then angle BAE + angle BEA must be 90° and BAE=45°.

..... **(3)**

Copyright©2013 Elite Learning Ltd

b) Work out length AB.



Looking at triangle ABO

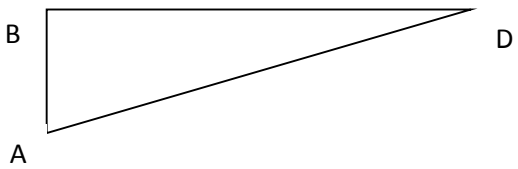
We know that AO is 5cm (half the diameter) and that angle BAO is 45° as found above. So we can use SOHCAHTOA.

We have the adjacent and we are finding the hypotenuse to

$\cos 45 = \frac{a}{h} = \frac{AO}{AB}$ and rearranging this we get that $AB = \frac{5}{\cos 45} = 5\sqrt{2} = 7.07$cm (3.s.f).

..... **(2)**

c) Work out length BD.



We are given that AD = 30cm and we know that ABD is a right angle. We have just worked out that AB is $5\sqrt{2}$ cm so we can use Pythagoras to find BD.

$AD^2 = BD^2 + AB^2$

$BD^2 = AD^2 - AB^2$ therefore $BD^2 = 30^2 - (5\sqrt{2})^2 = 900 - 50 = 850$ therefore

$BD = \sqrt{850} = 29.2\ (3.s.f)$ cm.

..... **(2)**

Question 8: TOTAL: /7

.....

Question 9.

There are only red balls and green balls in a bag.

There are 7 red balls and 5 green balls.

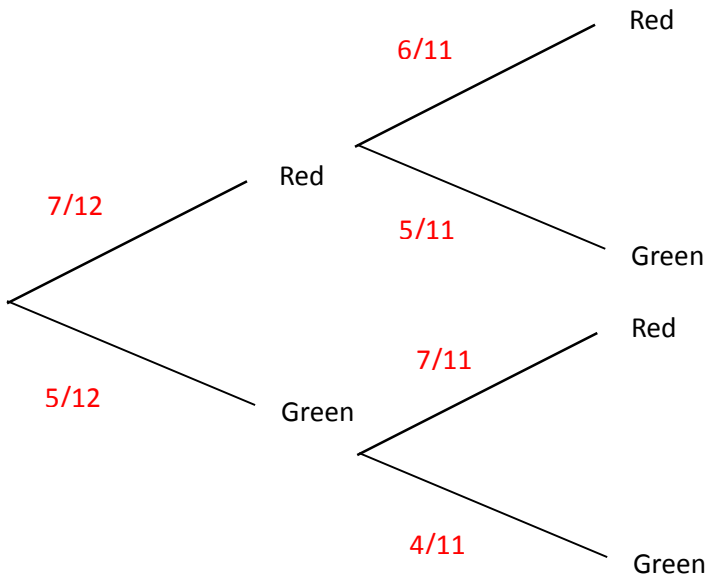
Dustin takes at random a ball from the bag.

He does not put the ball back in the bag.

Dustin takes at random a second ball from the bag.

a) Complete the probability tree diagram.

Copyright©2013 Elite Learning Ltd



(2)

b) Work out the probability that Dustin takes balls of different colours.

For Dustin to take a ball of different colours he can either take a Red ball followed by a Green ball OR a Green ball followed by a Red ball.

The probability of Red and then Green (by following the branch) we get $P(RG) = \frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$.

The probability of Green and then Red (by following the branch) we get $P(GR) = \frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$.

Therefore the probability of Dustin taking balls of different colours is $\frac{35}{132} + \frac{35}{132} = \frac{70}{132} = \frac{35}{66}$.

(2)

Question 9: TOTAL: /4

Copyright©2013 Elite Learning Ltd

Question 10.

a) Factorise fully

$$6a^2 + 24ab$$

Find things in common and bring them outside the brackets so $= 6a(a + 4b)$

..... **(2)**

b) Factorise

$$2x^2 - 5x - 3$$

This a quadratic so it needs to be separated into two brackets. To make the first term we need to start with $(2x \quad)(x \quad)$. To make the last term we have to have a 1 and a 3, one of which must be minus. By trial and error we find the middle terms $(2x \ + 1)(x - 3)$.

..... **(2)**

c) Simplify

$$\frac{x^2 - 5x + 4}{x - 4}$$

We need to first factorise the top which is a quadratic so we separate this into two brackets to get -

$$\frac{(x-4)(x-1)}{x-4} = x - 1$$

..... **(2)**

d) Simplify

$$2x^2y^2 \times 7x^3y^3$$

This is effectively $2 \times 7 \times x^2 \times x^3 \times y^2 \times y^5 = 14x^5y^5$

..... **(2)**

Question 10: TOTAL: /8

.....

Copyright©2013 Elite Learning Ltd

Question 11. Solve

$$\frac{3(2x-1)^2}{3x-2} = 4x - 1$$

The first thing to do is to get rid of the denominator on the left hand side by multiplying it up on the other side.

$$3(2x-1)^2 = (4x-1)(3x-2)$$

$$3(2x-1)(2x-1) = (4x-1)(3x-2)$$

Then multiple out each side $3(4x^2 - 4x + 1) = 12x^2 - 11x + 2$

$$12x^2 - 12x + 3 = 12x^2 - 11x + 2$$

The $12x^2$ cancel from both sides leaving $-12x + 3 = -11x + 2$

Rearranging gives $3 = -11x + 12x + 2$

So $3 = x + 2$

and therefore $x = 1.$

..... **(5)**

Question 11: TOTAL: /5

Copyright©2013 Elite Learning Ltd

Question 12.

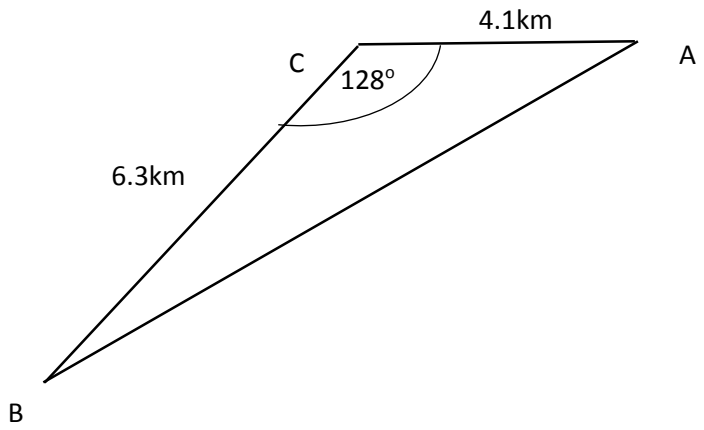


Diagram **NOT** accurately drawn

Here is a triangle ABC.

AC = 4.1 km.
BC = 6.3 km.
Angle *ACB* = 128°.

a) Calculate the perimeter of the triangle.
Give your answer correct to one decimal place.

The perimeter of the triangle is $P = BC + AC + AB$ and therefore we need to find length BA. As we have more lengths than angles and it is a scalene triangle we use the cos formula on the front of the exam paper.

$a^2 = b^2 + c^2 - 2bc\cos A = 6.3^2 + 4.1^2 - 2(6.3)(4.1)\cos 128 = 88.30507.$

Therefore $a = \sqrt{88.30507}$ = 9.40 km.

Therefore the perimeter $= 9.40 + 6.3 + 4.1 = 19.8$ (3.s.f) km. ..... **(4)**

b) Calculate the area of the triangle.
Give your answer correct to 2 significant figures.

The area of the triangle is given by $area = \frac{1}{2} \times 6.3 \times 4.1 \sin 128 = 10.177 = 10\ (2.s.f)\ km^2$.

(NOTE: be careful that use two lengths and the INCLUDED angle).

..... **(3)**

Question 12: TOTAL: /7

Copyright©2013 Elite Learning Ltd

Question 13.

Solve the equation

$$3x^2 - 12x - 7 = 0$$

Give each solution correct to 2 decimal places.

If the question asks you to give you answer to 2 decimal places then you are going to have to use the formula which is on the front of the exam paper.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 3, b = -12$ and -7. $x = \frac{12 \pm \sqrt{144} - 4(3)(-7)}{6}$

Therefore $x_1 = \frac{12 + \sqrt{144} - 4(3)(-7)}{6} = 4.52$ (2.d.p) and $x_2 = \frac{12 - \sqrt{144} - 4(3)(-7)}{6} = -0.52$ (2.d.p).

..... **(3)**

Question 13: TOTAL: /3

.....

Question 14.

Solve the simultaneous equations

$$3x - y = -2$$

$$3x + 4y = 18$$

To solve simultaneously you need to eradicate one of the unknowns.

If we subtract the 2nd equation from the first we will get $-y - 4y = -2 - 18$

$-5y = -20$ and therefore $y = 4$.

We then substitute this value back into the first equation to get $3x - 4 = -2$. Rearranging to get $3x = 2$ and therefore $x = \frac{2}{3}$.

x = ..., y = ... **(4)**

Question 14: TOTAL: /4

.....

Copyright©2013 Elite Learning Ltd

Question 15.

Make *p* the subject of the formula

$$\frac{6(p\ +\ q)}{p} + 3 =\ 5$$

Give your answer in its simplest form.

Start by multiplying up by p to get rid of the denominator

$$6(p+q)+3p=5p$$

Multiple out the brackets $6p+6q+3p=5p$

Take all the p's to the left and all the q's to the right we get $6p+3p-5p=-6q$

and so $4p=-6q$ and $p=-\frac{3}{2}q$.

p = ... **(3)**

Question 15: TOTAL: /3

.....

Question 16.

a) Write in standard form 78, 900, 000

These must be written with just one number in front of the decimal point.

So this is given as 7.89×10^{7}.

..... **(1)**

b) Write in standard form 0.0000028

This is given as 2.8×10^{-6}.

..... **(1)**

Question 16: TOTAL: /2

.....

Copyright©2013 Elite Learning Ltd

Question 17.

m is an integer such that $-5 \leq m \leq 1$

a) List all the possible values of *m*.

This means that *m* is greater than or equal to -5 and *m* is less than or equal to 1.

So *m* can take the values $-5, -4, -3, -2, -1, 0, 1$.

..... **(2)**

b) Solve the inequality

$$6x - 3 \geq x + 18$$

Start by moving the x's to the left handside and the numbers to the right $6x - x \geq 3 + 18$

$5x \geq 21$ and therefore $x \geq \frac{21}{5}$ and $x \geq 4\frac{1}{5}$ $(or\ 4.2)$.

..... **(2)**

Question 17: TOTAL: /4

.....

Question 18.

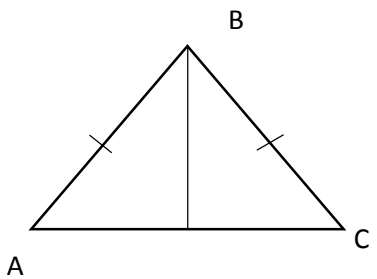


Diagram **NOT** accurately drawn

ABC is an isosceles triangle with base AC = a and height = b.

Where *a*= 9.13 cm correct to 2 decimal places and *b*= 5.23 cm correct to 2 decimal places.

Copyright©2013 Elite Learning Ltd

a) By considering bounds, work out the minimum area of triangle ABC.

You must show all your working and give a reason for your final answer.

The area of a triangle is given by = $\frac{1}{2} \times base \times height$. To work out the minimum area as the base and the height are multiplied we will need the minimum area of the base and minimum area of the height. The minimum area of the base, $a_{min} = 9.125$ cm and the minimum height, $b_{min} = 5.225$.

Therefore the minimum area is given by, $Area_{min} = \frac{1}{2} \times a_{min} \times b_{min} = \frac{1}{2} \times 9.125 \times 5.225 = 23.8\ (3.s.f)$.

Area = ...cm^2 **(3)**

b) By also consider bounds, work out the minimum perimeter of triangle ABC.

The minimum perimeter of the triangle ABC is $AB_{min} + BC_{min} + AC_{min}$ but as $AB = BC$ we need just to work out length AB.

If M is the midpoint of AC then triangle AMB is a right angled triangle where $AM_{min} = \frac{1}{2} \times 9.125 = 4.5625$. If we use Pythagora's theorem to find AB we get that

$AB_{min}^2 = b_{min}^2 + 4.5625^2 = 48.1170315$ and $AB = 6.936644091$.

Therefore the minimum perimeter of triangle ABC =$6.936644091 * 2 + 9.125 = 22.99828818 = 23.0\ cm\ (3.s.f)$.

Perimeter = ...cm **(3)**

Question 18: TOTAL: /6

Copyright©2013 Elite Learning Ltd

Question 19.

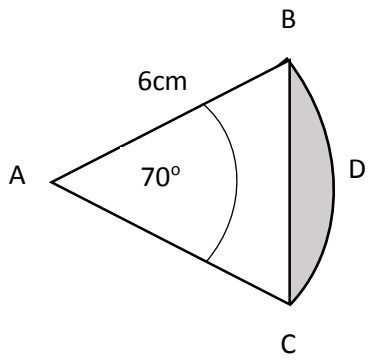


Diagram **NOT** accurately drawn

BAC is a sector of a circle, radius 6 cm.

The angle BAC is 70°.

a) Work out the length of BC.

As BAC is a sector of a circle AC is also 6cm. We can then use the formulas on the front of the exam paper for a scalene triangle to find BC. In this question there are more lengths than angles so we use the 'cos' formula i.e. $a^2 = b^2 + c^2 - 2bc\cos A$. So for this example

$BC^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \cos 70 = 47.3745$ and therefore $BC = 6.88\ (3.s.f)$.

(3)

b) Work out the perimeter of segment BDC.

The perimeter of segment BDC is equal to the length BC plus the length of the arc BDC.

The circumference of a circle with radius 6cm is given by $= 2 \times \pi \times r = 2 \times \pi \times 6 = 37.6991$.

But the length BDC is only a proportion of this. The exact proportion is $\frac{70}{360}$. Therefore the length of BDC $= \frac{70}{360} \times 37.6991 = 7.33$. The perimeter is therefore $= 6.88 + 7.33 = 14.2\ (3.s.f)$ cm .

(4)

Question 19: TOTAL: /7

Copyright©2013 Elite Learning Ltd

Question 20.

Show that $0.\dot{2}\dot{4}$ can be written as $\frac{24}{99}$.

If we let	$n = 0.\dot{2}\dot{4} = 0.242424242424 \ldots.$
If we multiple both sides by 100 we get	$100n = 24.242424242424 \ldots.$
If we then carry out do 100n –n we get	$100n - n = 24$
Therefore	$99n = 24$
And	$n = \frac{24}{99}$.

(4)

Question 20: TOTAL: /4

……

……

Copyright©2013 Elite Learning Ltd