1. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0$$

find $\frac{dy}{dx}$. **(4)**

1. Using nx^{n-1} $\qquad \frac{dy}{dx} = 12x^2 + 2(\frac{1}{2}x^{-1/2})$

Therefore $\qquad \frac{dy}{dx} = 12x^2 + x^{-1/2}$

2. a) Express √108 in the form *a*√3, where *a* is an integer. **(1)**
b) Express (2 - √3)^2 in the form *b+c*√3 , where *b* and *c* are integers to be found. **(3)**

a) √108 = √36 x √3 $\qquad$ =6√3

b) (2 - √3)(2 - √3) $\qquad$ = 4 – 2√3 – 2√3 +√3√3

=7 – 4√3

3. Graph question….

4. Solve the simultaneous equations

Eqn 1 $\qquad \boldsymbol{y = x - 2}$

Eqn 2 $\qquad \boldsymbol{y^2 + x^2 = 10}$ **(7)**

Substitute for *y* into eqn (2). $\qquad (x-2)(x-2) + x^2 = 10$

Expanding out the brackets $\qquad x^2 - 4x + 4 + x^2 = 10$

$2x^2 - 4x + 4 = 10$

$2x^2 - 4x - 6 = 0$

Divide by 2 $\qquad x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

Therefore $\qquad x = 3 \ or \ x = -1$

Substitute back into eqn (1) to find y — When $x = 3, y = 1$; When $x = -1, y = -3$

5. The equation $2x^2 - 3x - (k+1) = 0$, where *k* is a constant, has no real roots.

Find the set of possible values of k. **(4)**

For a quadratic equation in the form $ax^2 + bx + c = 0$ to have no Real roots $b^2 - 4ac < 0$.

Therefore $(-3)^2 - 4(2)(-k-1) = 0$

$9 + 8k + 8 < 0$

$8k + 17 < 0$

$k < -\frac{17}{8}$

6. a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$ where *k* is a constant to be found. **(2)**

b) Find $\int (4 + 3\sqrt{x})^2\, dx$ **(3)**

a) Expanding out $(4 + 3\sqrt{x})(4 + 3\sqrt{x}) = 16 + 12\sqrt{x} + 12\sqrt{x} + 9\sqrt{x}\sqrt{x}$

$= 16 + 24\sqrt{x} + 9x$

Therefore k=24

b) $\int 16 + 24\sqrt{x} + 9x\, dx = \int 16 + 24x^{1/2} + 9x\, dx$

Using $\frac{1}{n+1}x^{n+1}$ $= 16x + \frac{24}{3/2}x^{3/2} + \frac{9}{2}x^2 + C$

$= 16x + 16x^{3/2} + \frac{9}{2}x^2 + C$

7. The curve C has equation $y = f(x)$, $x \neq 0$, and the point *P(2,1)* lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2}$$

a) Find f(x) **(5)**

b) Find an equation for the tangent to C at the point P, giving your answer in the form *y=mx+c*, where *m* and *c* are integers. **(4)**

a)

$$f(x) = \int f'(x)dx$$

$$f(x) = \int 3x^2 - 6 - 8x^{-2}\,dx$$

$$f(x) = 3\left(\frac{1}{3}\right)x^3 - 6x - 8(-1)x^{-1} = x^3 - 6x + \frac{8}{x} + c$$

Substitute in P (2,1)

$$1 = 8 - 12 + 4 + C$$

C=1

Therefore

$$f(x) = x^3 - 6x + \frac{8}{x} + 1$$

$b)\ m = \frac{dy}{dx}$ for a tangent

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2}$$

$$at\ f'(x_p) = 3(2)^2 - 6 - \frac{8}{2^2} = 12 - 6 - 2 = 4$$

$$y = 4x + c$$

Substituting in for P(2,1)

$$1 = 8 + c$$

$$c = -7$$

Therefore

$$y = 4x - 7$$

8. The curve C has the equation $y = 4x + 3x^{3/2} - 2x^2$, $x > 0$.

a) Find an expression for $\frac{dy}{dx}$. **(3)**

b) Show the point P(4,8) lies on C. **(1)**

c) Show that an equation of the normal to C at the point P is $3y = x + 20$. **(4)**

The normal to C at P cuts the x-axis at the point Q.

d) Find the length PQ, giving your answer in a simplified surd form. **(3)**

Using nx^{n-1}

$$\frac{dy}{dx} = 4 + 3\left(\frac{3}{2}\right)x^{1/2} - 4x$$

$$\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$$

b) Substitute P(4,8) into the equation of the curve

$$8 = 4(4) + 3(4)^{3/2} - 2(4)^2$$

$$8 = 16 + 24 - 32$$

$$8 = 40 - 32$$ QED.

c) For the equation of the normal $\frac{dy}{dx} = -\frac{1}{m}$ at P

$$\frac{dy}{dx} = 4 + \frac{9}{2}(4)^{1/2} - 4(4)$$

$$\frac{dy}{dx} = 4 + 9 - 16 = -3$$

Therefore

$m = \frac{1}{3}$ and $y = \frac{1}{3}x + c$

Substitute P to find c.

$$8 = \frac{4}{3} + c$$

$$c = \frac{20}{3}$$

$$y = \frac{1}{3}x + \frac{20}{3}$$

Multiply by 3

$$3y = x + 20$$

d) Firstly finding Q at x-axis and so y=0

$$0 = x + 20, \quad and\ x = -20$$

Length PQ

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8 - 0)^2 + (4 - -20)^2}$$

$$= \sqrt{64 + 576 = \sqrt{640} = 8\sqrt{10}}$$

9. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1:
Row 2:
Row 3:

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

a) Find an expression, in terms of *n*, for the number of sticks required to make a similar arrangement of *n* squares in the *nth* row. (3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

b) Find the total number of sticks Ann uses in making these 10 rows. (3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete *k* rows but does not have sufficient sticks to complete the *(k+1)*th row,

c) Show that *k* satisfies $(3k - 100)(k + 35) < 0$. (4)

***d)* Find the value of *k*. (2)**

a) Using $u_n = a + (n-1)d$

As *a*=4 and *d*=3 $u_n = 4 + (n-1)3$

$u_n = 1 + 3n$

b) Using $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = 5(8 + 9(3)) = 5(35) = 175$

c) Using $S_n = \frac{n}{2}(2a + (n-1)d)$

$1750 > \frac{k}{2}(8 + (k-1)3)$

Multiply by 2 $3500 > k(5 + 3k)$

$0 > 3k^2 + 5k - 3500$

Factorising $0 > (3k - 100)(k + 35)$

d) Solving $k = -35 \text{ or } k = \frac{100}{3} = 33.3$

But *k>0* and need to be an integer. So k=33

10. a) On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x-2)$, (3)

(ii) $y = x(6-x)$, (3)

and indicate on your sketches the coordinates of all the points where the curves crosses the x-axis.

b) Use algebra to find the coordinates of the points where the graphs intersect. (7)

a) (i) standard x^3 with a double root at x=0 and one root at x=2. When x=0, y=0. Then plot graph.
(ii) Standard $-x^2$ curve (minus so mound) with two roots at x=0 and x=6. When x=0, y=0.

b) At intersect then the curves are equal to each other so

$x^2(x-2) = x(6-x)$
$x^3 - 2x^2 = 6x - x^2$
$x^3 - x^2 - 6x = 0$
$x(x^2 - x - 6) = 0$
$x(x-3)(x+2) = 0$
$x = 0, 3, -2$

Find y

$x = 0, y = 0$
$x = 3, y = 3(6-3) = 9$
$x = -2, y = -2(6+2) = -16$