1. a) Write down the value of $125^{1/3}$ **(1)**
b) Find the value of $125^{-2/3}$ **(2)**

a) $125^{1/3}$ $=\sqrt[3]{125}$

$=5$

b) $125^{-2/3}$ $=(\sqrt[3]{125})^{-2}$

$=(5)^{-2}$

$=\frac{1}{5^2} \quad = \frac{1}{25}$

2. Find $\int(12x^5 - 8x^3 - 3)\,dx$, giving each term in its simplest form. **(4)**

Using $\frac{1}{n+1}x^{n+1}$ $=\frac{12x^6}{6} - \frac{8x^4}{4} + 3x + c$

$= 2x^6 - 2x^4 + 3x + c$

3. Expand and simplify $(\sqrt{7}+2)(\sqrt{7}-2)$ **(2)**

$= \sqrt{7} \text{ x } \sqrt{7} + \cancel{2\sqrt{7}} - \cancel{2\sqrt{7}} - 4$

Leaving $7 - 4 = 3$

4. A curve has equation y = f(x) and passes through the point (4,22)
Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find *f(x)*, giving each term in its simplest form. **(5)**

$f(x) = \int f'(x)dx$

$= \int 3x^2 - 3x^{\frac{1}{2}} - 7\,dx$

Using $\frac{1}{n+1}x^{n+1}$ $$= \frac{3x^3}{3} - \left(\frac{3}{\frac{3}{2}}\right)x^{\frac{3}{2}} - 7x + c$$

$$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + c$$

Use *f(x)=22* when *x=4* to find c. $$22 = 4^3 - 2(4)^{\frac{3}{2}} - 7(4) + c$$

$$22 = 64 - 2(2)^3 - 7(4) + c$$

$$22 = 64 - 16 - 28 + c$$

$$2 = c$$

Therefore $$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + 2$$

5. Figure 1 shows a sketch of the curve *C* with equation *y* = f(*x*). There is a maximum at (0,0), a minimum at (2,-1) and *C* passes through (3,0).

On a separate diagram sketch the curve with equation

(a) y = f(x+3), (3)

(b) y = f(-x). (3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

a) Replace inside brackets from x to x+3 — $x + 3 = 0$ — x=-3 shift on x-axis

b) y=f(-x) — Reflection in the y-axis

6. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of *p* and the value of *q*. (2)

Given that $= 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$ simplifying the coefficient of each term. (4)

a)

$$\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}} = \frac{2x^2}{\sqrt{x}} - \frac{x^{\frac{3}{2}}}{\sqrt{x}}$$

$= 2x^2 . x^{-\frac{1}{2}} - x^{\frac{3}{2}} . x^{-\frac{1}{2}}$

$= 2x^{\frac{3}{2}} - x$

b) $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$

$y = 5x^4 - 3 + 2x^{\frac{3}{2}} - x$

Using nx^{n-1} $\quad \dfrac{dy}{dx} = 20x^3 + \left(\dfrac{3}{2}\right)(2)x^{\frac{1}{2}} - 1$

$\dfrac{dy}{dx} = 20x^3 + 3x^{\frac{1}{2}} - 1$

7. The equation $x^2 + 4x + (5 - k) = 0$, where *k* is a constant, has 2 different real solutions for *x*.

(a) Show that *k* satisfies

$$k^2 - 5k + 4 > 0$$ **(3)**

(b) Hence find the set of possible values of *k*. **(4)**

a) For 2 Real roots $\quad b = 4, a = k, c = 5 - l$

$b^2 - 4ac > 0$

Where $ax^2 + bx + c = 0$

Therefore $\quad 16 - 4(k)(5 - k) > 0$

$16 - 20k + 4k^2 > 0$

Divide by 4 $\quad 4 - 5k + k^2 > 0$

$k^2 - 5k + 4 > 0$

b) $\quad (k - 4)(k - 1) > 0$

Therefore critical values $\quad k = 4 \text{ or } k = 1$

As k^2 is a U shape $\quad$ This curve is > 0 *at* $k < 1$ *and* $k > 4$

8. The point *P*(1,*a*) lies on the curve with equation $y = (x + 1)^2(2 - x)$

(a) Find the value of *a* **(1)**

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x+1)^2(2-x)$

(ii) $y = \frac{2}{x}$

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}$$

(1)

a) $a = (1+1)^2(2-1) \qquad a = 4$

b) When y = 0, x = -1 or x = 2
Double root at x = -1 so is a turning point. The shape is $-x^3$ curve.

(c) The two curves only intersect twice in +y and +x quadrant ∴ there are only 2 ℝeal roots.

9. The first term of an arithmetic series is *a* and the common difference is *d*.

The 18th term of the series is 25 and the 21st term of the series is 32½.

(a) Use the information to write down two equations for *a* and *d*. **(2)**

(b) Show that *a* = -17.5 and find the value of *d*. **(2)**

The sum of the first *n* terms of the series is 2750.

(c) Show that *n* is given by

$$n^2 - 15n = 55 \times 4$$ **(4)**

(d) Hence find the value of *n*. **(3)**

a) Using $u_n = a + (n-1)d$

$u_{18} = a + 17d = 25 \quad (1)$
$u_{21} = a + 20d = 32.5 \quad (2)$

b) Solve simultaneous equations (1)-(2) $-3d = -7.5$
$d = 2.5$

Substitute into (1) $a + 17(2.5)d = 25$ a=-17.5

c) Using $S_n = \frac{n}{2}[2a + (n-1)d]$ $2750 = \frac{n}{2}[2(-17.5) + (n-1)2.5]$

X2 $5500 = n[-35 + 2.5n - 2.5]$

$5500 = n[-37.5 + 2.5n]$

X2 $11000 = n[-75 + 5n]$

$0 = -75n + 5n^2 - 11000$

Divide by 5 $0 = -15n + n^2 - 2200$

$0 = n^2 - 15n - (55 \text{ x } 40)$

$(55 \text{ x } 40) = n^2 - 15n$

$0 = n^2 - 15n - (55 \text{ x } 40)$

$0 = (n + 40)(n - 55)$ $n = -40, n = 55$ $but\ n > 0\ and\ n = 55$

10. The line l_1 passes through the point *A* (2,5) and has a gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$. **(3)**

The point *B* has coordinates (-2,7).

(b) Show that *B* lies on l_1. **(1)**

(c) Find the length of *AB*, giving your answer in the form $k\sqrt{5}$, where k is an integer. **(3)**

The point *C* lies on l_1 and has *x*-coordinate equal to *p*.
The length of AC is 5 units.

(d) Show that *p* satisfies

$$p^2 - 4p - 16 = 0.$$ **(4)**

a) Using $y = mx + c$ $y = -\frac{1}{2}x + c$

Substitute in (2,5)	$5 = -\frac{1}{2}2 + c$ $5 = -1 + c \quad c = 6$
Therefore	$y = -\frac{1}{2}x + 6$
b) Substitute in (-2,7)	$y = -\frac{1}{2}(-2) + 6 \quad y = 7 \quad QED$
c) Using length of line $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$= \sqrt{(7-5)^2 + (-2-2)^2}$ $= \sqrt{2^2 + 4^2}$ $= 2\sqrt{5}$
d) If p lies on l_1 then	$y = -\frac{1}{2}p + 6$
P (p, $-\frac{1}{2}p + 6$) = (x_1, y_1) & A (2,5) = (x_2, y_2)	$5 = \sqrt{\left(5-(-\frac{1}{2}p+6)\right)^2 + (2-p)^2}$ $5 = \sqrt{\left(\frac{1}{2}\text{p} - 1\right)^2 + 4 - 4\text{p} + \text{p}^2}$
Square both sides	$25 = \frac{1}{4}p^2 - \frac{1}{2}\text{p} - \frac{1}{2}\text{p} + 1 + 4 - 4\text{p} + \text{p}^2$
X4	$100 = p^2 - 2\text{p} - 2\text{p} + 20 - 16\text{p} + 4\text{p}^2$ $100 = 5p^2 - 20p + 20$ $5p^2 - 20p - 80 = 0$ $p^2 - 4p + 16 = 0$

11. The curve *C* has the equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point *P* on *C* has *x*-coordinate equal to 2.

(a) Show that the equation of the tangent to *C* at the point *P* is $y = 1 - 2x$. **(6)**

(b) Find an equation of the normal to *C* at the point *P*. **(3)**

The tangent *P* meets the *x*-axis at *A* and the normal at *P* meets the *x*-axis at *B*.

(c) **The area of the triangle *APB*.** **(4)**

a) Gradient of tangent is $\frac{dy}{dx}$	$\frac{dy}{dx} = -4 + 8x^{-2}$
At x = 2	$= -4 + \frac{8}{2^2} = -2$
Therefore	$y = -2x + c$
At x=2	$y = 9 - 4(2) - \frac{8}{2} = 9 - 8 - 4 = -3$
Put back in to find c	$y = -2x + c$
	$-3 = -4 + c \qquad c = 1$
Therefore	$y = -2x + 1$
	$y = 1 - 2x$
b) Equation of normal $-\frac{1}{m}$	$y = \frac{1}{2}x + c$
Using (2,-3) to find c	$-3 = \frac{2}{2} + c \qquad c = -4$
	$y = \frac{1}{2}x - 4$
c) l_1 meets *x*-axis at A. y = 0 at *x*-axis	$1 - 2x = 0 \qquad x = \frac{1}{2}$
l_2 meets *x*-axis at B. y = 0 at *x*-axis	$0 = \frac{1}{2}x - 4 \qquad x = 8$
Area of triangle $= \frac{1}{2}$(base) x (height)	$= \frac{1}{2}\left(8 - \frac{1}{2}\right) \times (y_p)$
	$= \frac{1}{2}\left(\frac{15}{2}\right)(-3) = \frac{45}{4}$