1. Simplify $(3+\sqrt{5})(3-\sqrt{5})$

(2)

1. Expand $= 9 + 3\sqrt{5} - 3\sqrt{5} - 5$

Therefore $= 9 - 5 = 4$

2. (a) Find the value of $8^{\frac{4}{3}}$

(2)

(b) Simplify $\frac{15x^{\frac{4}{3}}}{3x}$

(2)

a) $= (\sqrt[3]{8})^4 = 2^4 = 16$

b) $= 5x^{\frac{4}{3}}.x^{-1}$

$= 5x^{\frac{1}{3}}$

3. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, **find**

(a) $\frac{dy}{dx}$,

(2)

(b) $\frac{d^2y}{dx^2}$

(2)

(c) $\int y\,dx$

(3)

a) Rewrite equation $y = 3x^2 + 4x^{\frac{1}{2}}$

Using nx^{n-1}

$$\frac{dy}{dx} = 3.2x^1 + 4\frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$$

b) Differentiate again

$$\frac{d^2y}{dx^2} = 6 + 2.-\frac{1}{2}x^{-\frac{3}{2}} = 6 - x^{-\frac{3}{2}}$$

c) To integrate using $\frac{1}{n+1}x^{n+1}$

$$\int y\,dx = 3\frac{1}{3}x^3 + \frac{4}{3/2}x^{\frac{3}{2}} + c$$

$$= x^3 + \frac{8}{3}x^{\frac{3}{2}} + c$$

4. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3)

a) From the information given it is clear that a=5p and d=2p.

Using $u_n = a + (n-1)d$

$$u_{200} = 5 + 199 \times 2 = 403\text{p}$$

b) Using $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{200} = \frac{200}{2}[2 \times 5 + 199 \times 2]$$

$$S_{200} = 100[10 + 398] = 100 \times 408 = 40800p = £408$$

5. Graph question

6. By eliminating *y* from the equations

$$\boldsymbol{y = x - 4}$$
$$\boldsymbol{2x^2 - xy = 8}$$

show that

$$\boldsymbol{x^2 + 4x - 8 = 0}$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$\boldsymbol{y = x - 4}$$
$$\boldsymbol{2x^2 - xy = 8}$$

giving your answers in the form *a* ± *b*√3, where *a* and *b* are integers.

(5)

a) Substitute y into second equation

$$2x^2 - x(x-4) = 8$$

$$2x^2 - x^2 + 4x - 8 = 0$$

$$x^2 + 4x - 8 = 0$$

b) As there is a ± symbol this implies it does not factorise. Either complete the square or use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{4^2 + 4 \times 8}}{2} = \frac{-4 \pm \sqrt{48}}{2} = \frac{-4 \pm 4\sqrt{3}}{2}$$

$$x = -2 \pm 2\sqrt{3} \qquad a = -2 \text{ and } b = 2$$

Then find y $y = x - 4 = -6 \pm 2\sqrt{3}$

7. The equation $x^2 + kx + (k+3) = 0$, where k is a constant, has different real roots.
(a) Show that $k^2 - 4k - 12 > 0$ **(2)**

(b) Find the set of possible values of k. **(4)**

a) For different roots $b^2 - 4ac > 0$

$k^2 - 4(k+3) > 0$

$k^2 - 4k - 12 > 0$

b) Factorise $(k-6)(k+2) > 0$

Critical values are $k = 6 \; or \; k = -2$

As k^2 the curve is a U shape which is greater than 0 when $k < -2 \; and \; k > 6$

8. A sequence a_1, a_2, a_3 ... is defined by

$$a_1 = k$$
$$a_{n+1} = 3a_n + 5 \qquad n \geq 0$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. **(1)**

(b) Show that $a_3 = 9k + 20$. **(2)**

(c) (i) Find

$$\sum_{r=1}^{4} a_r$$

in terms of k.
(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10. **(4)**

a) Use $a_{n+1} = 3a_n + 5$ $a_2 = 3a_1 + 5 = 3k + 5$

b) Use $a_{n+1} = 3a_n + 5$ again $a_3 = 3(3k + 5) + 5$

$a_3 = 9k + 20$

c) $a_4 = 3(9k + 20) + 5 = 27k + 60 + 5 = 27k + 65$

$\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$

Therefore $= k + 3k + 5 + 9k + 20 + 27k + 65$

(i) $= 40k + 90$

(ii) Divisible by 10 $= \frac{40k + 90}{10} = 4k + 9 \ QED$

9. The curve *C* with equation *y* = f (*x*) passes through the point (5, 65).
Given that

$$f'(x) = 6x^2 - 10x - 12$$

(a) use integration to find *f (x)* . **(4)**
(b) Hence show that

$$f(x) = x(2x + 3)(x - 4).$$

(2)

(c) In the space provided on page 17, sketch *C*, showing the coordinates of the points where *C* crosses the *x*-axis.

(3)

9a) $f(x) = \int f'(x)\,dx$
Using
$\frac{1}{n+1}x^{n+1}$

$f(x) = 6\frac{1}{3}x^3 - 10\frac{1}{2}x^2 - 12x + c$

$f(x) = 2x^3 - 5x^2 - 12x + c$

Substitute P(5,65) to find

$65 = 2 \times 5^3 - 5 \times 5^2 - 12 \times 5 + c$
$65 = 250 - 125 - 60 + c \qquad c = 0$

Therefore $f(x) = 2x^3 - 5x^2 - 12x$

b) Bring out an x $f(x) = x(2x^2 - 5x - 12) = x(2x + 3)(x - 4)$

c) Curve C crosses the x-axis at $x = 0, x = -\frac{3}{2}, x = 4$

Plot using the traditional x^3 curve.

10. The curve *C* has equation

$$y = x^2(x - 6) + \frac{4}{x} \quad x > 0$$

The points *P* and *Q* lie on *C* and have *x*-coordinates 1 and 2 respectively.
(a) Show that the length of *PQ* is √170.

(4)

(b) Show that the tangents to *C* at *P* and *Q* are parallel.

(5)

(c) Find an equation for the normal to *C* at *P*, giving your answer in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.

(4)

a) Find y for x=1 and 2

$y = 1(1-6) + 4 = -1$

$y = 4(2-6) + 2 = -14$

Using $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(2-1)^2 + (-14 - -1)^2}$

$PQ = \sqrt{1 + 13^2} = \sqrt{170}$

b) The gradient of tangent is $\frac{dy}{dx} = m$

$y = x^3 - 6x^2 + 4x^{-1}$

$$\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$$

At x=1 $\quad \frac{dy}{dx} = 3 \times 1^2 - 12 - 4 = -13$

At x=2 $\quad \frac{dy}{dx} = 3 \times 2^2 - 24 - 1 = -13$

As the gradients are the same then the lines are parallel.

c) Gradient of normal $= -\frac{1}{m} = \frac{1}{13}$ $\quad y = mx + c$

$$y = \frac{1}{13}x + c$$

Sub in P(1,-1) $\quad -1 = \frac{1}{13} + c \qquad c = -\frac{14}{13}$

Therefore

$$y = \frac{1}{13}x - \frac{14}{13}$$

$$13y = x - 14$$

$$0 = x - 13y - 14$$

11. The line l_1 has equation *y* = 3*x* + 2 and the line l_2 has equation $+2y - 8 = 0$.
(a) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is *P*.
(b) Find the coordinates of *P*.

(3)

The lines l_1 and l_2 cross the line *y* = 1 at the points *A* and *B* respectively.
(c) Find the area of triangle *ABP*.

(4)

a) Rearrange to be y=

$$2y = -3x + 8$$

$$y = -\frac{3}{2}x + 4 \qquad and\ so\ m = -\frac{3}{2}$$

b) Solve simultaneously $-\frac{3}{2}x+4=3x+2$

X2 $-3x+8=6x+4$

$4=9x \quad x=\frac{4}{9}$

Find y $y=3x+2 \quad y=3\frac{4}{9}+2=\frac{10}{3}$

Coordinates are $P(\frac{4}{9},\frac{10}{3})$

c) When y=1 coordinates of A $1=3x+2 \quad x=-\frac{1}{3}$

When y=1 coordinates of B $3x+2-8=0 \quad 3x=6 \quad x=2$

Area of triangle

$$Area=\frac{1}{2}base\times height$$

Base is therefore $2\frac{1}{3}$

Height is y_p-1

Area $=\frac{1}{2}\left(\frac{7}{3}\right)\left(\frac{7}{3}\right)=\frac{49}{18}$