1. Simplify
(a) $(3\sqrt{7})^2$

(1)

(b) $(8+\sqrt{5})(2-\sqrt{5})$

(3)

a) $(3\sqrt{7})^2 = 9 \times 7 = 63$

b) Expand out $(8+\sqrt{5})(2-\sqrt{5}) = 16 - 8\sqrt{5} + 2\sqrt{5} - 5 = 11 - 6\sqrt{5}$

2. Given that $32\sqrt{2} = 2^a$, find the value of a. **(3)**

$32\sqrt{2} = 2^5 . 2^{\frac{1}{2}} = 2^{\frac{11}{2}}$ $\quad a = \frac{11}{2}$

3. Given that $y = 2x^3 + \frac{3}{x^2}$, $\quad x \neq 0$
Find
(a) $\frac{dy}{dx}$

(3)

(b) $\int y\,dx$**, simplifying each term.**

(3)

a) Re-write as follows $y = 2x^3 + 3x^{-2}$

nx^{n-1} $\quad \frac{dy}{dx} = 6x^2 - 6x^{-3}$

b) Using $\frac{1}{n+1}x^{n+1}$ $\quad \int 2x^3 + \frac{3}{x^2}dx = \frac{2}{4}x^4 + \frac{3}{-1}x^{-1} + C$

$= \frac{1}{2}x^4 - 3x^{-1} + C$

4. Find the set of values of *x* for which
(a) $4x - 3 > 7 - x$ **(2)**
(b) $2x^2 - 5x - 12 < 0$ **(4)**
(c) both $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$ **(1)**

a) $4x - 3 > 7 - x$

$5x > 7 + 3 \quad 5x > 10 \quad x > 2$

b) Factorise $(2x+3)(x-4) < 0$ so the critical values are

$x = 4 \; and \; x = -\frac{3}{2}$

The curve is positive so is a U shape therefore this curve is < 0 between the factors.

$-\frac{3}{2} < x < 4$

c) $2 < x < 4$

5. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term *a* and common difference *d*.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of *d*, **(3)**
(b) the value of *a*, **(2)**
(c) the total number of houses built in Oldtown over the 40-year period. **(3)**

a) 1960 is the 10th year

Using

$u_n = a + (n-1)d$

$2400 = a + 9d \quad (1)$

$600 = a + 39d \quad (2)$

(2)-(1) $\quad -1800 = 30d \qquad d = -60$

b) Put back into (1) $\quad 2400 = a + 9(-60) \qquad a = 2940$

c) Using

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[5880 + 39 \times -60] = 20[3540] = 70800$

6. The equation $x^2 + 3px + p = 0$, where *p* is a non-zero constant, has equal roots. Find the value of *p*.

(4)

For equal roots

$b^2 - 4ac = 0$

$(3p)^2 - 4p = 0$

$9p^2 - 4p = 0$

$p(9p - 4) = 0 \qquad p = 0 \; or \; p = \frac{4}{9} \qquad p \neq 0 \; so \; p = \frac{4}{9}$

7. A sequence $a_1, a_2, a_3 \ldots$..is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geq 1$$

where *k* is a constant.

(a) Write down an expression for a_2 in terms of *k*. **(1)**

(b) Show that $a_3 = 4k - 21$. **(2)**

Given that

$$\sum_{r=1}^{4} a_r = 43$$

(c) find the value of *k*.

(4)

a) Put a_1 into formula $\quad a_2 = 2a_1 - 7 = 2k - 7$

b) Put a_2 into formula $\quad a_3 = 2(2k - 7) - 7 = 4k - 21$

c) Find a_4 and then add them all $\quad a_4 = 2(4k - 21) - 7 = 8k - 49$

Therefore

$$\sum_{r=1}^{4} a_r = 43 = k + 2k - 7 + 4k - 21 + 8k - 49$$

$$43 = 15k - 77 \qquad 15k = 120 \qquad k = 8$$

8. The points *A* and *B* have coordinates (6, 7) and (8, 2) respectively.

The line *l* passes through the point *A* and is perpendicular to the line *AB*, as shown in Figure 1.

(a) Find an equation for *l* in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.

(4)

Given that *l* intersects the *y*-axis at the point *C*, find

(b) the coordinates of *C*,

(2)

(c) the area of Δ*OCB*, where *O* is the origin.

(2)

a) The gradient AB is $\quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-7}{8-6} = \frac{-5}{2}$

Therefore the gradient of the line perpendicular to AB is $\quad = -\frac{1}{5/2} = \frac{2}{5}$

Using $\quad y = \frac{2}{5}x + c$

$y = mx + c$

Put in the co-ordinates of A $\quad 7 = \frac{12}{5} + c \qquad c = \frac{23}{5}$

Therefore $y = \frac{2}{5}x + \frac{23}{5}$

Rearrange to the form they request. $5y = 2x + 23$

$0 = 2x - 5y + 23$

b) At y axis x=0. $0 = -5y + 23 \qquad y = \frac{23}{5}$

Coordinates of C are therefore $(0, \frac{23}{5})$

c) Area of OCB

$Area = \frac{1}{2} base \times height$

Where base = OC= $\frac{23}{5}$.

And height is x coordinates of B = 8

Area $= \frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$

9.

$$\boldsymbol{f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}}$$

(a) Show that $\boldsymbol{f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B}$ where *A* and *B* are constants to be found. **(3)**

(b) Find $\boldsymbol{f'(x)}$. **(3)**

(c) Evaluate $\boldsymbol{f'(9)}$. **(2)**

9 a) Multiply out the top

$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}} = \frac{(3 - 4\sqrt{x})(3 - 4\sqrt{x})}{\sqrt{x}}$$

$$f(x) = \frac{9 - 24\sqrt{x} + 16x}{\sqrt{x}} = \frac{9}{\sqrt{x}} - 24\frac{\sqrt{x}}{\sqrt{x}} + 16\frac{x}{\sqrt{x}}$$

$$f(x) = 9x^{-\frac{1}{2}} - 24 + 16x^{\frac{1}{2}} \qquad A = 16, B = -24$$

b) Using nx^{n-1}

$$f'(x) = \frac{dy}{dx} = 9\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} + 16\frac{1}{2}x^{-\frac{1}{2}}$$
$$= -\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}$$

c) Find $f'(9)$ just put 9 in.

$$f'(9) = -\frac{9}{2}(9)^{-\frac{3}{2}} + 8(9)^{-\frac{1}{2}} = -\frac{9}{2}(3)^{-3} + 8(3)^{-1}$$
$$= -\frac{9}{2}.\frac{1}{27} + \frac{8}{3} = -\frac{1}{6} + \frac{8}{3} = \frac{-1 + 16}{6} = \frac{15}{6} = \frac{5}{2}$$

10. (a) Factorise completely

$$x^3 - 6x^2 + 9x$$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the *x*-axis.

(4)

Using your answer to part (b), or otherwise,
(c) sketch, on a separate diagram, the curve with equation

$$y = (x-2)^3 - 6(x-2)^2 + 9(x-2)$$

showing the coordinates of the points at which the curve meets the *x*-axis.

(2)

10a. Bring x out

$$x^3 - 6x^2 + 9x = x(x^2 - 6x + 9)$$

$$x^3 - 6x^2 + 9x = x(x-3)^2$$

b. Standard x^3 curve with a root at x=0 and a double root (i.e. the curve turns at x=3.
c. This represents f(x-2) which is a shift of 2 along the x-axis. Therefore there is a single root x=2 and a double root and turning point at x=5.

11. The curve *C* has equation

$$y = x^3 - 2x^2 - x + 9, \qquad x > 0$$

The point *P* has coordinates (2, 7).

(a) Show that *P* lies on *C*. **(1)**

(b) Find the equation of the tangent to *C* at *P*, giving your answer in the form *y* = *mx* + *c*, where *m* and *c* are constants.
(5)

The point *Q* also lies on *C*. Given that the tangent to *C* at *Q* is perpendicular to the tangent to *C* at *P*,

(c) show that the *x*-coordinate of *Q* is

$$= \frac{1}{3}(2 + \sqrt{6})$$

(5)

11 a. Put P into C	$y = 2^3 - 2.2^2 - 2 + 9 = 8 - 8 - 2 + 9 = 7 = y$ QED.
b. Using y=mx+c where $\frac{dy}{dx} = m$	$\frac{dy}{dx} = 3x^2 - 4x - 1$
At P, x=2	$\frac{dy}{dx} = 3 \times 2^2 - 4 \times 2 - 1 = 12 - 8 - 1 = 3$
Fill in (2,7) to find c	$y = 3x + c$
	$7 = 3 \times 2 + c \qquad c = 1$
	$y = 3x + 1$
If perpendicular to P then gradient at $Q = -\frac{1}{3}$. Therefore	$3x^2 - 4x - 1 = -\frac{1}{3}$
Multiple by 3	$9x^2 - 12x - 3 = -1$
	$9x^2 - 12x - 2 = 0$
Using $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$	$x = \frac{12 \pm \sqrt{(144 - 4 \times 9 \times -2)}}{18} = \frac{12 \pm \sqrt{216}}{18}$
	$x = \frac{12 \pm \sqrt{216}}{18} = \frac{12 \pm 6\sqrt{6}}{18} = \frac{2 \pm \sqrt{6}}{3}$
We are told x>0 therefore	$x = \frac{1}{3}(2 + \sqrt{6})$