1. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) *x* – 3,

(ii) *x* + 2.

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

(4)

1a) (i) Substitute in x=3 $3^3 - 2 \times 3^2 - 4 \times 3 + 8 = 27 - 18 - 12 + 8 = 5$

(i) Substitute in x=-2 $2^3 - 2 \times 2^2 - 4 \times 2 + 8 = 8 - 8 - 8 + 8 = 0$

b) As x+2 is a factor, factorise as follows.

$$\begin{array}{r} x^2 - 4x + 4 \\ (x+2)\sqrt{x^3 - 2x^2 - 4x + 8} \\ \underline{x^3 + 2x^2} \quad \text{(subtract)} \\ -4x^2 - 4x + 8 \\ \underline{-4x^2 - 8x} \quad \text{(subtract)} \\ 4x + 8 \\ \underline{4x + 8} \quad \text{(subtract)} \\ 0 \end{array}$$

The factors are $x^3 - 2x^2 - 4x + 8 = 0 = (x+2)(x^2 - 4x + 4)$

$= (x+2)(x-2)^2$

Therefore x=2 or x=-2. There is a double root at x=2.

2. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

2 a) Using $u_n = ar^{n-1}$ $10 = ar^3$ (1)

$80 = ar^6$ (2)

Divide (2) by (1) $\frac{80}{10} = \frac{ar^6}{ar^3}$ $r^3 = 8$ $r = 2$

b) Substitute into (1) $10 = 1$ $a = 1.25$

c) Using

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{20} = \frac{1.25(1-2^{20})}{1-2} = 1.25(2^{20}-1)$$

$$S_{20} = 1310718.75 = 1310719 \ (nearest\ whole\ number)$$

3. (a) Find the first 4 terms of the expansion of $(1+\frac{x}{2})^{10}$ in ascending powers of *x*, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

3 a) Using

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} +$$

$$(1+\frac{x}{2})^{10} = 1 + 10\frac{x}{2} + \frac{10 \times 9}{2}(\frac{x}{2})^2 + \frac{10 \times 9 \times 8}{6}\left(\frac{x}{2}\right)^3 \ldots.$$

$$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 \ldots.$$

b) Observe that $(1.005)^{10}$ can be written as $(1+0.005)^{10}$ then by comparison

$$\frac{x}{2} = 0.005 \quad x = 0.01$$

Sub this into the expansion

$$= 1 + 5(0.01) + \frac{45}{4}(0.01)^2 + 15(0.01)^3 \ldots.$$

$$= 1.05114 \ (5.d.p)$$

4. (a) Show that the equation

$$\mathbf{3\,sin^2\,\theta - 2\,cos^2\,\theta = 1}$$

can be written as

$$\mathbf{5\,sin^2\,\theta = 3.}$$

(2)

(b) Hence solve, for $0° < \theta < 360°$, the equation

$$\mathbf{3\,sin^2\,\theta - 2\,cos^2\,\theta = 1}$$

giving your answers to 1 decimal place.

(7)

4 a) Using

$$cos^2x + sin^2x = 1$$

$$3\,sin^2\,\theta - 2\,(1 - sin^2x) = 1$$

$$5\,sin^2\,\theta - 2 = 1.$$

$$5\,sin^2\,\theta = 3.$$

b) Using the answer to part a)

$$\sin^2\theta = \frac{3}{5} \qquad \sin\theta = \pm\sqrt{(\frac{3}{5})}$$

Start with $\sin\theta = +\sqrt{(\frac{3}{5})}$

$$\theta_1 = \sin^{-1}\left(\sqrt{\frac{3}{5}}\right) = 50.8° \ (1.d.p)$$

By observation of the sin curve it is apparent that

$$\theta_2 = 180 - \theta_1 = 129.2° \ (1.d.p)$$

Also $\sin\theta = -\sqrt{\frac{3}{5}}$

$$\theta_3 = 180 + \theta_1 = 230.8° \ (1.d.p)$$

$$\theta_4 = 360 - \theta_1 = 309.2° (1.d.p)$$

5. Given that *a* and *b* are positive constants, solve the simultaneous equations

$$\boldsymbol{a = 3b,}$$
$$\boldsymbol{\log_3 a + \log_3 b = 2.}$$

Give your answers as exact numbers.

(6)

5. Sub first into second

$$\log_3 3b + \log_3 b = 2.$$

Using $\log a + \log b = \log ab$

$$\log_3 3b^2 = 2$$

Anti-logging

$$3b^2 = 3^2 \qquad b = \pm\sqrt{3}$$

But b>0 (in question) so

$$b = \sqrt{3} \ and \ a = 3\sqrt{3}$$

6. Figure 1 shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane.

Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from *A*. The bearing of *C* from *A* is 015°.

(a) Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures.

(3)

The bearing of yacht *C* from yacht *B* is *ϑ*°, as shown in Figure 1.
(b) Calculate the value of *ϑ*.

(4)

6 a) Using the cos formula $a^2 = b^2 + c^2 - 2bc \cos A$

$$a^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15°$$

$$a^2 = 63851.9216$$
$$a = 253 \ m \ \ (3.s.f)$$

b) Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{700 \sin 15}{252.69} = sinB$

$0.716978 = sinB$
$45.81 = B$

7. a) At x-axis y=0 $\quad 0 = 6x - x^2 = x(6-x) \quad x = 0 \text{ and } x = 6$

b) At L solve simultaneously $\quad 2x = 6x - x^2 \quad x^2 - 4x = x(x-4)$

$x = 0 \text{ or } x = 4$

When x=0 $\quad y = 2x = 0 \quad Coordinates\ (0,0)$

When x=4 $\quad y = 2x = 8 \quad Coordiantes\ (4,8)$

c) Area of R= Area under curve minus Area of triangle.

Area under curve

$$\int_0^4 y\,dx = \int_0^4 6x - x^2\,dx = \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^4$$

$$= \frac{6 \times 16}{2} - \frac{64}{3} - 0 = 26\frac{2}{3}$$

Area of Triangle $\quad Area = \frac{1}{2} base \times height = \frac{1}{2} \times 4 \times 8 = 16$

Therefore Area of R $\quad R = 26\frac{2}{3} - 16 = 10\frac{2}{3}$

8. A circle *C* has centre *M* (6, 4) and radius 3.
(a) Write down the equation of the circle in the form

$$(\mathbf{x} - \mathbf{a})^2 + (\mathbf{y} - \mathbf{b})^2 = \boldsymbol{r}^2$$

(2)

Figure 3 shows the circle *C*. The point *T* lies on the circle and the tangent at *T* passes through the point *P* (12, 6). The line *MP* cuts the circle at *Q*.

(b) Show that the angle *TMQ* is 1.0766 radians to 4 decimal places.

(4)

The shaded region *TPQ* is bounded by the straight lines *TP*, *QP* and the arc *TQ*, as shown in Figure 3.

(c) Find the area of the shaded region *TPQ*. Give your answer to 3 decimal places.

(5)

8a) Sub in centre and radius $\quad (x-6)^2 + (y-4)^2 = 3^2 = 9$

b) Use trig first to find PM $\quad PM = \sqrt{((12-6)^2 + (6-4)^2)} = \sqrt{40}$

	$cos\theta = \frac{3}{\sqrt{40}}$ $\theta = cos^{-1}\frac{3}{\sqrt{40}} = 1.0766\, rads$
c) Area of TPQ=Area of TMP – Area of sector	Area of TMP= $\frac{1}{2}(TP) \times 3$
	$\sin(1.7066) = \frac{TP}{\sqrt{40}}$ $TP = 5.57$
	Area of TMP= 8.352
Area of sector using $Area = \frac{1}{2}r^2\theta$	Area of TMP= $\frac{1}{2} \times 3^2 \times (1.0766) = 4.8447$
Therefore	Area of TPQ=8.352 - 4.8447= 3.507 (3.d.p)

9. Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle *x* metres by *y* metres. The height of the tank is *x* metres.

The capacity of the tank is 100 m^3.

(a) Show that the area *A* m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2$$

(4)

(b) Use calculus to find the value of *x* for which *A* is stationary.

(4)

(c) Prove that this value of *x* gives a minimum value of *A*.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

9 a) Use volume to eliminate y	$Vol = x \times x \times y = 100$ $y = \frac{100}{x^2}$
Put y into surface area	$A = 2x^2(sides) + 2xy(sides) + xy(base)$
	$A = 2x^2 + 3xy$
	$A = 2x^2 + 3x\frac{100}{x^2} = \frac{300}{x} + 2x^2$
b) At stationary point $\frac{dA}{dx} = 0$	$A = 300x^{-1} + 2x^2$
Using nx^{n-1}	$\frac{dA}{dx} = -1(300)x^{-2} + 4x = 0$
	$\frac{300}{x^2} = 4x$ $x^3 = \frac{300}{4}$ $x = \sqrt[3]{\frac{300}{4}} = 4.22$
c) For a minimum value $\frac{d^2A}{dx^2} > 0$	$\frac{d^2A}{dx^2} = -1 \times -2 \times (300)x^{-3} + 4 = \frac{600}{x^3} + 4 > 0$
d) Find A for x=4.2171 by putting in formula	$A = \frac{300}{4.2171} + 2(4.2171)^2$

$$A = 106.71\ cm^2$$