**1. Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of $(3 - 2x)^5$ , giving each term in its simplest form.**

**(4)**

1. Bring the 3 out as it must start with a 1

Using

$$(3 - 2x)^5 = 3^5(1 - \frac{2}{3}x)^5$$

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

$$3^5(1 - \frac{2}{3}x)^5 = 3^5(1 + 5\left(-\frac{2}{3}x\right) + \frac{5 \times 4}{2}(-\frac{2}{3}x)^2 \ldots$$

$$= 3^5\left(1 - \frac{10}{3}x + 10 \times \frac{4}{9}x^2 \ldots\right)$$

$$= 243 - 810x + 1080x^2 \ldots$$

**2. Figure 1 shows part of the curve *C* with equation**

$$\boldsymbol{y = (x + 1)(4 - x)}$$

**The curve intersects the *x*-axis at *x* = −1 and *x* = 4. The region *R*, shown shaded in Figure 1, is bounded by *C* and the *x*-axis.**

**Use calculus to find the exact area of *R*.**

**(5)**

2. Area under the curve is

$$\int_a^b y\,dx$$

Using

$$\frac{1}{n+1}x^{n+1}$$

$$\int_{-1}^{4}(x+1)(4-x)dx = \int_{-1}^{4} 4x + 4 - x - x^2\,dx$$

$$= \int_{-1}^{4} 3x + 4 - x^2\,dx = \left[\frac{3}{2}x^2 + 4x - \frac{x^3}{3}\right]_{-1}^{4}$$

$$= \left[24 + 16 - \frac{64}{3}\right] - \left[\frac{3}{2} - 4 + \frac{1}{3}\right]$$

$$= \frac{125}{6}$$

**3.**

$$\boldsymbol{y = \sqrt{(10x - x^2)}}$$

**(a) Complete the table below, giving the values of *y* to 2 decimal places.**

| | | | | | | |
|---|---|---|---|---|---|---|
| **x** | **1** | **1.4** | **1.8** | **2.2** | **2.6** | **3** |
| **y** | **3** | **3.47** | **3.84** | **4.14** | **4.39** | **4.58** |

**(2)**

**(b) Use the trapezium rule, with all the values of *y* from your table, to find an approximation for the value of**

$$\int_1^3 \sqrt{10x - x^2}dx$$

**(4)**

3a) Simply put value of x into formula for y

b) Using

$$\int_1^3 \sqrt{10x - x^2}dx \approx \frac{h}{2}\{y_0 + y_n + 2(y_1 + y_2 \ldots\ldots y_{n-1})\}$$

H is the band width = 1.4-1=0.4

$$\approx \frac{0.4}{2}\{3 + 4.58 + 2(3.47 + 3.84 + 4.14 + 4.39\}$$
$$\approx 0.2(39.26)$$
$$\approx 7.852\ (4.s.f)$$

**4. Given that 0 < *x* < 4 and**

$$\boldsymbol{log_5(4 - x) - 2log_5 x = 1}$$

**find the value of *x*.**

**(6)**

4. Using $alog_b c = log_b c^a$

$$log_5(4 - x) - log_5 x^2 = 1$$

Using $loga - logb = log\frac{a}{b}$

$$log_5 \frac{(4 - x)}{x^2} = 1$$

Anti-logging

$$\frac{(4 - x)}{x^2} = 5^1 \qquad 4 - x = 5x^2$$

$$5x^2 + x - 4 = 0$$
$$(5x - 4)(x + 1) = 0$$

So

$$x = \frac{4}{5}\ or\ x = -1$$

But in the question we are told 0 < *x* < 4 therefore

$$x = \frac{4}{5}$$

**5. The points *P*(−3, 2), *Q*(9, 10) and *R*(*a*, 4) lie on the circle *C*, as shown in Figure 2. Given that *PR* is a diameter of *C*,**

**(a) show that *a* = 13,**

**(3)**

**(b) find an equation for *C*.**

**(5)**

5 a) Using Pythagoras. First find PQ

$$PQ = \sqrt{((9 - -3)^2 + (10 - 2)^2)}$$

| | |
|---|---|
| | $PQ = \sqrt{12^2 + 8^2}$ $\quad PQ^2 = 144 + 64 = 2 - 8$ |
| | $PR^2 = PQ^2 + QR^2$ |
| | $(a - -3)^2 + (4 - 2)^2 = 208 + (a - 9)^2 + (4 - 10)^2$ |
| | $a^2 + 6a + 9 + 4 = 208 + a^2 - 18a + 81 + 36$ |
| | $24a = 81 + 36 + 208 - 13$ |
| | $24a = 312$ |
| | $a = 13$ |
| b) Centre is the midpoint of PR | $M\left(\frac{13-3}{2}, \frac{4+2}{2}\right) = M(5,3)$ |
| c) Radius is the length PM (P(-3,2), M(5,3) | $PM = \sqrt{((5 - -3)^2 + (3 - 2)^2)}$ |
| | $PM = \sqrt{65}$ |
| Therefore equation of circle is | $(x - 5)^2 + (y - 3)^2 = 65$ |

**6.**

$$\boldsymbol{f(x) = x^4 + 5x^3 + ax + b,}$$

**where *a* and *b* are constants.**

**The remainder when f(*x*) is divided by (*x* – 2) is equal to the remainder when f(*x*) is divided by (*x* + 1).**

**(a) Find the value of *a*.**

**(5)**

**Given that (*x* + 3) is a factor of f(*x*),**

**(b) find the value of *b*.**

**(3)**

| | |
|---|---|
| 6 a) Put x=2 in and the x=-1 | $f(2) = 2^4 + 5(2)^3 + 2a + b$ |
| | $f(-1) = (-1)^4 + 5(-1)^3 - a + b$ |
| Make them equal | $2^4 + 5(2)^3 + 2a + b = (-1)^4 + 5(-1)^3 - a + b$ |
| | $16 + 40 + 2a = 1 - 5 - a$ |
| | $3a = -60$ |
| | $a = -20$ |
| b) Using factor theorem f(-3)=0 | $f(-3) = (-3)^4 + 5(-3)^3 - 20(-3) + b = 0$ |
| | $f(-3) = 81 - 135 + 60 + b = 0$ |
| | $b = -6$ |
| Therefore | $f(x) = x^4 + 5x^3 - 20x - 6$ |

**7. The shape *BCD* shown in Figure 3 is a design for a logo.**

**The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm. The size of ∠*BAC* is 2.2 radians and *AD* = 4 cm.**

**Find**

**(a) the area of the sector *BAC*, in cm$^2$,**

**(2)**

**(b) the size of ∠*DAC*, in radians to 3 significant figures,**

**(2)**

**(c) the complete area of the logo design, to the nearest cm$^2$.**

**(4)**

| | |
|---|---|
| 7 a) Using $Area = \frac{1}{2}r^2\theta$ | $Area = \frac{1}{2}6^2(2.2) = 39.6\ cm^2$ |
| b) Observe $DAC + DAB + BAC = 2\pi$ | And DAC=DAB |
| Therefore | $2DAC + BAC = 2\pi$<br>$2DAC + 2.2 = 2\pi$<br>$DAC + 2.2 = \pi - 1.1 = 2.04\ radians$ |
| c) Total Area =2 x Area of DAC+Area of Sector | Area = $2\frac{1}{2}ab\sin C + 39.6$<br>Area = $6 \times 4 \times \sin 2.04 + 39.6$<br>Area = $61\ cm^2$ |

**8. (a) Show that the equation**

$$\mathbf{4\,sin^2\,x + 9\,cos\,x - 6 = 0},$$

**can be written as**

$$\mathbf{4\,cos^2\,x - 9\,cos\,x + 2 = 0}.$$

**(2)**

**(b) Hence solve, for $\mathbf{0 < x < 720°}$,**

$$\mathbf{4\,sin^2\,x + 9\,cos\,x - 6 = 0},$$

**giving your answers to 1 decimal place.**

**(6)**

| | |
|---|---|
| 8 a) Using $cos^2x + sin^2x = 1$ | $4\,(1 - cos^2\,x) + 9\cos x - 6 = 0$<br>$4 - 4cos^2x + 9\cos x - 6 = 0$<br>$4\,cos^2\,x - 9\cos x + 2 = 0$ |
| b) Using proof of part a) and factorise using u=cosx | $4\,u^2 - 9u + 2 = 0$ |

| | |
|---|---|
| | $(4u-1)(u-2)=0 \qquad u=\frac{1}{4}\ or\ u=2$ |
| Therefore | $cosx=\frac{1}{4}\ or\ cosx=2\ but\ cosx\neq 2\ so\ cosx=\frac{1}{4}\ \ only$ |
| In the region $0 < x < 720°$ | $x_1=cos^{-1}\left(\frac{1}{4}\right)=75.5\ (1.d.p)$ |
| By observation of the cos curve | $x_2=360-75.52=284.5\ (1.d.p)$ |
| | $x_3=360+75.52=435.5\ (1.d.p)$ |
| | $x_4=720-75.52=644.5\ (1.d.p)$ |

**9. The first three terms of a geometric series are (*k* + 4), *k* and (2*k* – 15) respectively, where *k* is a positive constant.**

**(a) Show that $k^2 - 7k - 60 = 0$.**

**(4)**

**(b) Hence show that *k* = 12.**

**(2)**

**(c) Find the common ratio of this series.**

**(2)**

**(d) Find the sum to infinity of this series.**

**(2)**

| | |
|---|---|
| 9 a) Using $u_n=ar^{n-1}$ | $a=k+4\ \ (1)$ |
| | $ar=k\ \ (2)$ |
| | $ar^2=2k-15\ \ (3)$ |
| From (2) and (1) | $r=\frac{k}{a}=\frac{k}{k+4}$ |
| Substitute into 3 | $(k+4)(\frac{k}{k+4})^2=2k-15$ |
| | $\frac{k^2}{k+4}=2k-15$ |
| | $k^2=(k+4)(2k-15)$ |
| | $k^2=2k^2+8k-15k-60$ |
| | $0=k^2-7k-60$ |
| b) Factorise | $0=(k-12)(k+5)\quad k=12\ or\ k=-5$ |
| But in question k>0 therefore | $k=12$ |
| c) | $r=\frac{k}{k+4}=\frac{12}{16}=\frac{3}{4}$ |
| d) Using $S_\infty=\frac{a}{1-r}$ | $S_\infty=\frac{k+4}{1-\frac{3}{4}}=\frac{16}{1/4}=64$ |

**10. A solid right circular cylinder has radius *r* cm and height *h* cm.**
**The total surface area of the cylinder is 800 cm$^2$.**

**(a) Show that the volume, V cm$^3$, of the cylinder is given by**

$$\boldsymbol{V = 400r - \pi r^3}.$$

**(4)**

**Given that r varies.**

**(b) Use calculus to find the maximum value of V, to the nearest cm$^3$.** **(6)**

**(c) Justify that the value of V you have found is a maximum.** **(2)**

| | |
|---|---|
| 10 a) Use the surface area to get rid of h. | Surface is $800 = 2 \times \pi r^2 + 2\pi rh$ |
| Rearrange for h | $800 - 2\pi r^2 = 2\pi rh$<br>$\dfrac{800 - 2\pi r^2}{2\pi r} = h$ |
| Substitute into volume | Volume $= \pi r^2 h = \pi r^2 \left(\frac{800-2\pi r^2}{2\pi r}\right) = \frac{r}{2}(800 - 2\pi r^2)$<br>$V = 400r - \pi r^3$ |
| b) At the maximum value $\frac{dV}{dr} = 0$. Using $nx^{n-1}$. | $\dfrac{dV}{dr} = 400 - 3\pi r^2 = 0$ |
| Rearrange for r | $3\pi r^2 = 400$<br>$r = \sqrt{\dfrac{400}{3\pi}}$ |
| Find V for this r | $V = 400\sqrt{\dfrac{400}{3\pi}} - \pi(\sqrt{\dfrac{400}{3\pi}})^3 = 1737$ |
| (c) To justify a maximum then $\frac{d^2V}{dr^2} < 0$. | $\frac{d^2V}{dr^2} = -6\pi r < 0$ as this is clearly negative. |
| Always include the sentence that says | Therefore this is a maximum. |