1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(4)

1. Bring the 3 out as the binomial must start with a 1

$$(3 - x)^6 = 3^6(1 - \frac{x}{3})^6$$

Using

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

$$3^6(1 - \frac{x}{3})^6 = 3^6(1 + 6\left(-\frac{x}{3}\right) + \frac{6 \times 5}{2}(-\frac{x}{3})^2)$$

$$= 3^6\left(1 - 2x + \frac{15}{9}x^2 \ldots..\right)$$

$$= 729 - 1458x + 1215x^3 \ldots.$$

2. (a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(2)

(b) Solve, for $0 < x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(4)

2a) Using
$\cos^2 x + \sin^2 x = 1$

$$\cos^2 x = 1 - \sin^2 x$$

Therefore

$$5 \sin x = 1 + 2(1 - \sin^2 x)$$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

b) Let u=sinx

$$2u^2 + 5u - 3 = 0$$

Factorise

$$(2u - 1)(u + 3) = 0$$

$$u = \frac{1}{2} \text{ or } u = -3$$

Therefore

$$sinx = \frac{1}{2} \text{ or } sinx \neq -3 \text{ (as not in range)}$$

In region for $0 < x < 360°$,

$$x_1 = \sin^{-1}\frac{1}{2} = 30°$$

From the curve you can see that

$$x_2 = 180 - x_1 = 150°$$

3.

$$f(x) = 2x^3 + ax^2 + bx - 6,$$

where *a* and *b* are constants.

When f(*x*) is divided by (2*x* – 1) the remainder is –5.
When f(*x*) is divided by (*x* + 2) there is no remainder.

(a) Find the value of *a* and the value of *b*.

(6)

(b) Factorise f(*x*) completely.

(3)

3a) Sub in $x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) - 6 = -5$$

$$f\left(\frac{1}{2}\right) = \frac{2}{8} + \frac{a}{4} + \frac{b}{2} - 6 = -5$$

Multiple by 4 $\quad 1 + a + 2b - 4 = 0$

$a + 2b - 3 = 0 \quad (1)$

Sub in x=-2 $\quad f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) - 6 = 0$

$f(-2) = -16 + 4a - 2b - 6 = 0$

$4a - 2b - 22 = 0 \quad (2)$

Do (1)+(2) $\quad 5a - 25 = 0 \qquad a = 5$

Sub back into (1) $\quad 5 + 2b - 3 = 0 \qquad 2 + 2b = 0 \qquad b = -1$

Therefore $\quad f(x) = 2x^3 + 5x^2 - x - 6$

b) As x+2 has no remainder it is a factor.

$$\begin{array}{r} 2x^2 + x - 3 \\ (x+2)\overline{)2x^3 + 5x^2 - x - 6} \\ \underline{2x^3 + 4x^2} \quad \text{(subtract)} \\ x^2 - x - 6 \\ \underline{x^2 + 2x} \quad \text{(subtract)} \\ -3x - 6 \\ \underline{-3x - 6} \quad \text{(subtract)} \\ 0 \end{array}$$

Therefore $\quad f(x) = (x + 2)(2x^2 + x - 3)$

And factorise the quadratic $\quad f(x) = (x + 2)(2x + 3)(x - 1)$

4. An emblem, as shown in Figure 1, consists of a triangle *ABC* joined to a sector *CBD* of a circle with radius 4 cm and centre *B.* The points *A*, *B* and *D* lie on a straight line with *AB* = 5 cm and *BD* = 4 cm. Angle *BAC* = 0.6 radians and *AC* is the longest side of the triangle *ABC*.

(a) Show that angle *ABC* = 1.76 radians, correct to 3 significant figures.

(4)

(b) Find the area of the emblem.

(3)

4a) Find ACB then do π-ACB - 0.6rad. Using $\frac{sinA}{a} = \frac{sinC}{c}$

$$\frac{sin0.6}{4} = \frac{sinC}{5}$$

$$\sin C = \frac{5sin0.6}{4}$$

C=0.784

$ABC = \pi - 0.6 - 0.784 = 1.76\ (3.s.f)$

b) Area of emblem is areas of triangle plus area of sector. Area of triangle.

$$Area = \frac{1}{2}ab\ sinC = \frac{1}{2} \times 5 \times 4\sin(1.76)$$

$Area = 9.82$

Area of sector $Area = \frac{1}{2}r^2\theta$

$Where\ \theta = \pi - 1.76$

$$Area = \frac{1}{2}4^2(1.76) = 11.0527$$

Total Area $Total\ Area\ =\ 9.82\ +\ 11.05\ = 20.87\ cm^2$

5. (a) Find the positive value of *x* such that

$$\boldsymbol{log_x 64\ =\ 2}$$

(2)

(b) Solve for *x*

$$\boldsymbol{log_2(11 - 6x)\ =\ 2\,log_2\,(x - 1)\ +\ 3}$$

(6)

5a) Using if $log_a b = c$ then $a^c = b$

$x^2 = 64 \qquad x = 8$

b) Using $alog_b c = log_b c^a$

$$log_2(11 - 6x)\ =\ log_2\,(x - 1)^2\ +\ 3$$

Using $loga - logb = log\frac{a}{b}$

$$log_2\frac{(11 - 6x)}{(x-1)^2} = 3$$

Anti-logging

$$\frac{(11 - 6x)}{(x-1)^2} = 2^3$$

$$(11 - 6x) = 8(x-1)^2$$

$$8x^2 - 10x - 3 = 0$$

$$(4x+1)(2x-3) = 0$$

Therefore

$$x = -\frac{1}{4}, x = \frac{3}{2}$$

6. A car was purchased for £18 000 on 1st January. On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.

(1)

The value of the car falls below £1000 for the first time *n* years after it was purchased.

(b) Find the value of *n*.

(3)

An insurance company has a scheme to cover the maintenance of the car.

The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

(2)

(d) Find the total cost of the insurance scheme for the first 15 years.

(3)

6 a) Geometric series with a=18000 and r=0.8

$$u_n = ar^{n-1} = 18000 \times (0.8)^2 = 9216$$

b) Using $u_n = ar^{n-1}$

$$1000 > 18000(0.8)^n$$
$$\frac{1}{18} > (0.8)^n$$
$$log_{0.8}\frac{1}{18} > n$$

n=12.95 so n=13 as must be integer.

c) Using a=200 and r=1.12

$$u_3 = 200(1.12)^2 = £314.70$$

d) Using

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{15} = \frac{200(1-r^{15})}{1-r} = £7455.94$$

7. The curve *C* has equation $y = x^2 - 5x + 4$. It cuts the *x*-axis at the points *L* and *M* as shown in Figure 2.

(a) Find the coordinates of the point *L* and the point *M*.

(2)

(b) Show that the point *N* (5, 4) lies on *C*.

(1)

(c) Find $\int (x^2 - 5x + 4)dx$.

(2)

The finite region *R* is bounded by *LN*, *LM* and the curve *C* as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of *R*.

(5)

7a) At x-axis y=0 therefore

$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0 \qquad x = 4\ or\ x = 1$$

So $L(1,0) \quad and \quad M(4,0)$

b) Substitute values into C to show they work $y = 5^2 - 5(5) + 4 = 25 - 25 + 4 = 4 \quad QED$

c) Using $\frac{1}{n+1}x^{n+1}$

$$\int x^2 - 5x + 4\,dx = \frac{x^3}{3} - \frac{5}{2}x^2 + 4x + c$$

d) Area of R is Area of triangle minus area under curve.

Area of triangle $= \frac{1}{2}(5-1)4 = 8$

Area under curve

$$= \left[\frac{x^3}{3} - \frac{5}{2}x^2 + 4x\right]_4^5$$

$$= \left[\frac{125}{3} - \frac{125}{2} + 20\right] - \left[\frac{64}{3} - \frac{5(16)}{2} + 4(4)\right]$$

$$= \frac{11}{6}$$

Area of Region R

$$= 8 - \frac{11}{6} = \frac{37}{6}$$

8. Figure 3 shows a sketch of the circle *C* with centre *N* and equation

$$(x-2)^2 + (y+1)^2 = \frac{169}{4}$$

(a) Write down the coordinates of *N*.

(2)

(b) Find the radius of *C*.

(1)

The chord *AB* of *C* is parallel to the *x*-axis, lies below the *x*-axis and is of length 12 units as shown in Figure 3.

(c) Find the coordinates of *A* and the coordinates of *B*.

(5)

(d) Show that angle *ANB* = 134.8°, to the nearest 0.1 of a degree.

(2)

The tangents to *C* at the points *A* and *B* meet at the point *P*.

(e) Find the length *AP*, giving your answer to 3 significant figures.

(2)

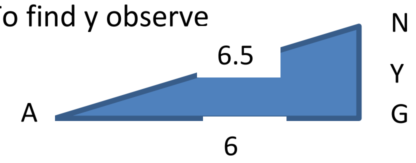
8 a) Compare with $(x-a)^2 + (y-b)^2 = r^2 \quad With\ centre(a,b)\ radius\ r$

Then a=2 and b=-1 Coordinates are N(2,-1)

b) By comparison $r^2 = \frac{169}{4} \qquad r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$

c) By observation of triangle ANB x coordinate of A = 2-6=-4 and B=2+6=8

To find y observe



$6.5^2 = 6^2 + y^2 \qquad y = 2.5$

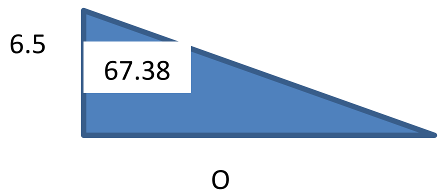
So y of A and B $\quad y = -1 - 2.5 = -3.5$

So A(-4,-3.5), B(8,-3.5)

d) Observe ANB=2ANG $\quad sin\theta = \frac{6}{6.5} \qquad \theta = sin^{-1}\frac{6}{6.5} \qquad \theta = 67.38°$

ANB= $2 \times 67.38 = 134.76 = 134.8°$

e) Observe a right angle triangle



Therefore $\quad tan67.38 = \frac{o}{6.5} \qquad o = 6.5tan67.38 \qquad o = 15.6 = AP$

9. The curve *C* has equation

$$y = 12\sqrt{x} - x^{\frac{3}{2}} - 10, \qquad x > 0$$

(a) Use calculus to find the coordinates of the turning point on *C*.

(7)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(c) State the nature of the turning point.

(1)

9a) At turning point $\frac{dy}{dx} = 0$ $\quad y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10$

Using nx^{n-1} $\quad \frac{dy}{dx} = 12\frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

Multiply by 2 $\quad 12x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} = 0$

$12x^{-\frac{1}{2}} = 3x^{\frac{1}{2}} \qquad \frac{12}{\sqrt{x}} = 3\sqrt{x} \qquad 12 = 3x \qquad x = 4$

When x=4 $\quad y = 12\,4^{\frac{1}{2}} - 4^{\frac{3}{2}} - 10 = 24 - 8 - 10 = 6$

Coordinates $\quad$ (4,6)

b) Differentiate again to find $\frac{d^2y}{dx^2}$ $\quad \frac{d^2y}{dx^2} = 6\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} - \frac{3}{2} \times \frac{1}{2}x^{-\frac{1}{2}} = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$

At turning point x=4 $\quad \frac{d^2y}{dx^2} = -3(4)^{-\frac{3}{2}} - \frac{3}{4}(4)^{-\frac{1}{2}} = -\frac{3}{8} - \frac{3}{8} = -\frac{6}{8} = -\frac{3}{4} < 0$

If $\frac{d^2y}{dx^2} < 0$ the turning point is a maximum

(Note if $\frac{d^2y}{dx^2} > 0$ the turning point would be a minimum).