1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x})dx$$

1.

$$= \int_1^4 \left(2x + 3x^{\frac{1}{2}}\right) dx$$

Using $\frac{1}{n+1}x^{n+1}$ (don't include a C if you have limits as they would cancel)

$$= \left[2\frac{1}{2}x^2 + \frac{3}{3/2}x^{\frac{3}{2}}\right] = \left[x^2 + 2x^{\frac{3}{2}}\right] = (4^2 + 2(2)^3) - (1 + 2)$$

$$= 32 - 3 = 29$$

2. (a) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

$$(2 + kx)^7$$

where *k* is a constant. Give each term in its simplest form. **(4)**

Given that the coefficient of *x*^{2} is 6 times the coefficient of *x*,
(b) find the value of *k*. **(2)**

a) Bring 2 out to make the expansion begin with a 1.

$$= 2^7(1 + \frac{kx}{2})^7$$

Using $(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$

$$= 2^7(1 + 7\left(\frac{kx}{2}\right) + \frac{7 \times 6}{2}\left(\frac{kx}{2})^2\right) \ldots\ldots..$$

$$= 128 + 448kx + 672k^2x^2 \ldots.$$

b) Therefore

$$448 \times 6k = 672k^2$$

$$\frac{2688}{672} = k \qquad k = 4$$

3.

$$f(x) = (3x - 2)(x - k) - 8$$

where *k* is a constant.
(a) Write down the value of f (*k*). **(1)**

When f (*x*) is divided by (*x* – 2) the remainder is 4

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(b) Find the value of *k*. **(2)**

(c) Factorise f (*x*) completely. **(3)**

a) Put k in for x $f(k) = (3k - 2)(k - k) - 8 = -8$

b) Therefore $f(2) = 4$ $f(2) = 4(2 - k) - 8 = -4k = 4 \quad k = -1$

c) $f(x) = (3x - 2)(x + 1) - 8$

$$f(x) = 3x^2 - 2x + 3x - 2 - 8$$

$$f(x) = 3x^2 + x - 10$$

$$f(x) = (3x - 5\ )(x + 2\ \ )$$

4 (a) Complete the table below, giving values of $\sqrt{(2^X + 1)}$ to 3 decimal places.

x	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
$\sqrt{(2^X + 1)}$	**1.414**	**1.554**	**1.732**	**1.957**	**2.236**	**2.580**	**3**

(2)

Figure 1 shows the region *R* which is bounded by the curve with equation *y* = √ (2*x* + 1), the *x*-axis and the lines *x* = 0 and *x* = 3

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of *R*. **(4)**

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of *R*. **(2)**

a) Filled in table

b) Using

$$y \approx \frac{h}{2}\{y_0 + y_n + 2(y_1 + y_2 \ldots\ldots y_{n-1})\}$$

Where h is 0.5-0=0.5

$$y \approx \frac{0.5}{2}\{1.414 + 3 + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)\} = 6.133$$

c) Overestimate as the trapezium strips are all above the curve and so it will be larger than the actual value.

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5. The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$ **(2)**

(b) Find the first term of the sequence. **(2)**

(c) Find the sum of the first 15 terms of the sequence. **(3)**

(d) Find the sum to infinity of the sequence. **(2)**

a) Using $u_n = ar^{n-1}$

Set up two equations

$$324 = ar^2 \quad (1)$$
$$96 = ar^5 \quad (2)$$

Do (2)÷(1)

$$\frac{96}{324} = r^3 \qquad r^3 = \frac{8}{27} \qquad r = \frac{2}{3}$$

b) Fill r back into (1)

$$324 = a\left(\frac{2}{3}\right)^2 \qquad 324 = \frac{4a}{9} \qquad a = 729$$

c) Using

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{15} = \frac{729\left(1-\frac{2}{3}^{15}\right)}{1-\frac{2}{3}} = 2182.0\ (1.d.p)$$

d) Using

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{729}{1-\frac{2}{3}} = 2187$$

6. The circle *C* has equation

$$\boldsymbol{x^2 + y^2 - 6x + 4y = 12}$$

(a) Find the centre and the radius of *C*. **(5)**

The point *P*(–1, 1) and the point *Q*(7, –5) both lie on *C*.

(b) Show that *PQ* is a diameter of *C*. **(2)**

The point *R* lies on the positive *y*-axis and the angle *PRQ* = 90°.

(c) Find the coordinates of *R*. **(4)**

a) Complete the square for x and y

$$\mathrm{x}^2 + \mathrm{y}^2 - 6\mathrm{x} + 4\mathrm{y} - 12 = (\mathrm{x}-3)^2 + (\mathrm{y}+2)^2 + \mathrm{k} = 0$$

$$\mathrm{x}^2 + \mathrm{y}^2 - 6\mathrm{x} + 4\mathrm{y} - 12 = \mathrm{x}^2 - 6\mathrm{x} + 9 + \mathrm{y}^2 + 4\mathrm{y} + 4 + \mathrm{k} = 0$$

Cancel terms

$$-12 = 9 + 4 + k \qquad k = -25$$

Therefore

$$(\mathrm{x}-3)^2 + (\mathrm{y}+2)^2 - 25 = 0$$

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$$(x-3)^2 + (y+2)^2 = 5^2$$

Compare with equation of a circle

$$(x-a)^2 + (y-b)^2 = r^2 \qquad With\ centre(a,b)\ radius\ r$$

Therefore

Centre (3,-2) and radius 5.

b) PQ is a diameter if it has length 10 and P and Q lie on C

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-5-1)^2 + (7--1)^2}$$

$$= \sqrt{100} = 10$$

Put Q into C

$$(7-3)^2 + (-5+2)^2 = 16+9 = 5^2$$

Put P into C

$$(-1-3)^2 + (1+2)^2 = 16+9 = 5^2$$

c) As PRQ is 90 and PQ is a diameter R has to lie on C and x=0. Therefore

$$(x-3)^2 + (y+2)^2 = 5^2$$

$$9 + (y+2)^2 = 5^2$$

$$(y+2)^2 = 16$$

$$y+2 = \pm 4 \qquad y = 2, -6 \quad y > 0 \quad y = 2$$

7. (i) Solve, for $-180° \leq \theta < 180°$,

$$\boldsymbol{(1 + tan\theta)(5sin\theta - 2) = 0}$$

(4)

(ii) Solve, for $0 \leq x < 360°$

$$\boldsymbol{4sinx = 3tanx}$$

(6)

(I)Take each bit in turn

$$(1 + tan\theta) = 0$$

For tan you can add 180 degress or subtract 180 degrees indefinitely.

$$tan\theta = -1 \qquad \theta = -45°, 135°$$

For sin in this region there is another result at 180 minus original value.

$$(5sin\theta - 2) = 0 \qquad sin\theta = \frac{2}{5} \qquad \theta = 23.58°, 156.42$$

Therefore

$$\theta = -45°, 135°, 23.58°, 156.42°$$

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(ii) Re-arramge and using $tanx = \frac{sinx}{cosx}$

$4sinx - 3\frac{sinx}{cosx} = 0 \qquad sinx\left(4 - \frac{3}{cosx}\right) = 0$

Take each bit in turn

$sinx = 0 \qquad x = 0, 180°$

For cos the next value is 360 minus the first value.

$3 = 4cosx \qquad x = cos^{-1}\frac{3}{4} \qquad x = 41.4°, 318.6°$

Therefore

$x = 0, 180°, 41.4°, 318.6°$

8. (a) Find the value of *y* such that

$$\boldsymbol{log_2 y = -3}$$

(2)

(b) Find the values of *x* such that

$$\boldsymbol{\frac{log_2 32 + log_2 16}{log_2 x} = log_2 x}$$

(5)

a) Anti-logging

$y = 2^{-3} = \frac{1}{8}$

b) Multiply up.

$log_2 32 + log_2 16 = (log_2 x)^2$

$log_2 32 + log_2 16 = (log_2 x)^2$

$5 + 4 = (log_2 x)^2$

$9 = (log_2 x)^2$

$\pm 3 = log_2 x$

Anti-logging

$x = 2^3, 2^{-3} = 8, \frac{1}{8}$

9. Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height *h* cm. The cross section is a sector of a circle. The sector has radius *r* cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, *S* cm^2, is given by

$$\boldsymbol{S = r^2 + \frac{1800}{r}}$$

(5)

(b) Use calculus to find the value of *r* for which *S* is stationary. **(4)**

(c) Prove that this value of *r* gives a minimum value of *S*. **(2)**

(d) Find, to the nearest cm^2, this minimum value of *S*. **(2)**

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a) Use the Volume to get rid of h	$Vol = Area\ of\ base \times height$
Area of sector $Area = \frac{1}{2}r^2\theta$	$300 = \frac{1}{2}r^2 \times 1 \times h \qquad h = \frac{600}{r^2}$
The surface area	$S = 2 \times \frac{1}{2}r^2 \times 1 + 2rh + r\vartheta h$ $S = r^2 + 2rh + rh$
Fill in h	$S = r^2 + \frac{1800}{r} = r^2 + 1800r^{-1}$
b) Stationary value at $\frac{dS}{dr} = 0$	$\frac{dS}{dr} = 2r - 1800r^{-2} = 0 \qquad 2r = \frac{1800}{r^2}$ $r^3 = 900 \qquad r = \sqrt[3]{900}$
c) If this is a minimum then $\frac{d^2S}{dr^2} > 0$	$\frac{d^2S}{dr^2} = 2 + 3600r^3 > 0\ so\ is\ a\ minimum$
d) Fill value of r into S	$S = 900^{\frac{2}{3}} + \frac{1800}{900^{\frac{1}{3}}} = 279.65cm^2$

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