1. (a) By writing $\sin 3\theta = \sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

(5)

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

1a) Using $\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$
Using $\sin 2x = 2\sin x\cos x$	$\sin(2\theta+\theta) = 2\sin\theta\cos^2\theta + \cos 2\theta\sin\theta$
Using $\cos 2x = 1 - 2\sin^2 x$	$\sin(2\theta+\theta) = 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$
And $\cos^2 x = 1 - \sin^2 x$	$\sin(2\theta+\theta) = 2\sin\theta(1-\sin^2\theta) + (1-2\sin^2\theta)\sin\theta$
	$\sin(2\theta+\theta) = 3\sin\theta - 4\sin^3\theta$
b) $\sin\theta = \frac{\sqrt{3}}{4}$	$\sin(3\theta) = 3\frac{\sqrt{3}}{4} - 4\frac{(3)^{\frac{3}{2}}}{4^3} = \frac{(3)^{\frac{3}{2}}}{4} - \frac{(3)^{\frac{3}{2}}}{16} = \frac{3(3)^{\frac{3}{2}}}{16} = \frac{(3)^{\frac{5}{2}}}{16} = \frac{9\sqrt{3}}{16}$

2

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2$$

(a) Show that

$$f(x) = \frac{x^2+x+1}{(x+2)^2}, \quad x \neq -2$$

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq 2$.

(1)

2a) Find a common denominator

$$f(x) = \frac{1(x+2)^2 - 3(x+2) \quad + 3}{(x+2)^2}$$

$$f(x) = \frac{x^2 + 4x + 4 - 3x - 6 \quad + 3}{(x+2)^2}$$

$$f(x) = \frac{x^2+x+1}{(x+2)^2}$$

b) Complete the square $x^2+x+1=(x+\frac{1}{2})^2+\frac{3}{4}$

Therefore as $(x+\frac{1}{2})^2>0\ and\ \frac{3}{4}>0\ then\ x^2+x+1>0$

c) As $(x+2)^2\ is\ always\ >0$

Therefore $\frac{x^2+x+1}{(x+2)^2}>0$

3. The curve *C* has equation

$$\boldsymbol{x\ =\ 2\sin y.}$$

(a) Show that the point $P(\sqrt{2},\frac{\pi}{4})$ lies on *C*.

(1)

(b) Show that $\frac{dy}{dx}=\frac{1}{\sqrt{2}}$ at *P*.

(4)

(c) Find an equation of the normal to *C* at *P*. Give your answer in the form *y* = *mx* + *c*, where *m* and *c* are exact constants.

(4)

3a) Put the values into RHS $x\ =\ 2\sin y=2sin\frac{\pi}{4}=\frac{2\sqrt{2}}{2}=\sqrt{2}=x\ QED$

b) Find $\frac{dx}{dy}$ $\frac{dx}{dy}\ =\ 2\cos y$

Therefore $\frac{dy}{dx}\ =\ \frac{1}{2\cos y}=\frac{1}{2}secy=\frac{1}{2}\times\frac{2}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

c) Using gradient of normal is $\frac{dy}{dx}=-\frac{1}{m}$ $y\ =\ -\sqrt{2}x+\ c$

Put P into curve $P(\sqrt{2},\frac{\pi}{4})$ $\frac{\pi}{4}=-\sqrt{2}\times\sqrt{2}+c\qquad c=\frac{\pi}{4}+2$

Therefore $y\ =\ -\sqrt{2}x+\frac{\pi}{4}+2$

4. (i) The curve *C* has equation

$$\boldsymbol{y=\frac{x}{9+x^2}}$$

Use calculus to find the coordinates of the turning points of *C*.

(6)

(ii) Given that

$$\boldsymbol{y=(1+e^{2x})^{\frac{3}{2}}}$$

find the value of $\frac{dy}{dx}\ at\ x=\frac{1}{2}ln3$.

(5)

4(i) At turning point $\frac{dy}{dx} = 0$

$$y = \frac{x}{9+x^2} = x(9+x^2)^{-1}$$

Use differentiation by parts

$u\frac{dv}{dx} + v\frac{du}{dx}$

$$u = x \quad \frac{du}{dx} = 1 \qquad v = (9+x^2)^{-1} \quad \frac{dv}{dx} = -2x(9+x^2)^{-2}$$

$$\frac{dy}{dx} = x.-2x(9+x^2)^{-2} + (9+x^2)^{-1}$$

$$\frac{dy}{dx} = \frac{-2x^2+9+x^2}{(9+x^2)^2} = 0 \qquad -x^2+9 = 0$$

$$x^2 = 9 \qquad x \pm 3$$

When x=3

$$y = \frac{x}{9+x^2} = \frac{3}{9+9} = \frac{3}{18} = \frac{1}{6}$$

When x=-3

$$y = \frac{x}{9+x^2} = \frac{-3}{9+9} = -\frac{3}{18} = -\frac{1}{6}$$

b) Find the value of $\frac{dy}{dx}$.
Differentiate the whole and then in the brackets.

$$y = (1+e^{2x})^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{\frac{1}{2}}.2e^{2x}$$

At $x = \frac{1}{2}ln3$.

$$\frac{dy}{dx} = \frac{3}{2}(1+e^{2\frac{1}{2}ln3})^{\frac{1}{2}}.2e^{2\frac{1}{2}ln3}$$

$$\frac{dy}{dx} = 3(1+3)^{\frac{1}{2}}.3 = 9\sqrt{4} = 18$$

5. Figure 1 shows an oscilloscope screen.
The curve shown on the screen satisfies the equation

$$y = \sqrt{3}cosx + sinx$$

(a) Express the equation of the curve in the form $y = Rsin(x+\alpha)$, where R and α are constants, $R > 0$, $and\ 0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Find the values of *x*, $0 \le x \le 2\pi$, for which *y* = 1.

(4)

5a) Compare with
$\sin(x+\alpha) = sinx\ cos\alpha + cosxsin\ \alpha$

$$sin\alpha = \sqrt{3}$$
$$cos\alpha = 1$$
$$R^2 = 3+1 \qquad R = 2$$

$$tan\alpha = \frac{sin\alpha}{cos\alpha} = \sqrt{3} \qquad \alpha = \frac{\pi}{3}$$

$$y = \sqrt{3}cosx + sinx = 2sin(x + \frac{\pi}{3})$$

b) When y=1

$2sin\left(x + \frac{\pi}{3}\right) = 1$

Let $p = x + \frac{\pi}{3}$ the domain of p is $\frac{\pi}{3} \le p \le 2\pi + \frac{\pi}{3}$

$sinp = \frac{1}{2} \quad p = sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ (not in range)

$$p_1 = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad and\ p_2 = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$$

Therefore

$$x_1 = \frac{5\pi}{6} - \frac{\pi}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$x_2 = \frac{13\pi}{6} - \frac{\pi}{3} = \frac{11\pi}{6}$$

6. The function f is defined by

$$\boldsymbol{f: x \to \ln(4 - 2x),\ \ x < 2\ \ and\ x \in \mathbb{R}}$$

(a) Show that the inverse function of f is defined by

$$\boldsymbol{f^{-1}: x \to 2 - \frac{1}{2}e^{x}}$$

and write down the domain of $f^{-1}: x$.

(4)

(b) Write down the range of $f^{-1}: x$.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the *x* and *y* axes.

(4)

The graph of *y* = *x* + 2 crosses the graph of $y = f^{-1}(x)$ at *x* = *k*.

The iterative formula

$$\boldsymbol{x_{n+1} = -\frac{1}{2}e^{x_n},\quad x_0 = -0.3}$$

is used to find an approximate value for *k*.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of *k* to 3 decimal places.

(2)

6a) exchange y and x and then make y the subject.

$y = \ln(4 - 2x) \qquad x = ln(4 - 2y)$

$e^x = 4 - 2y$

$2y = 4 - e^x \quad y = 2 - \frac{1}{2}e^x$

Domain is unrestricted so — Domain $f^{-1}(x) \; x \in \mathbb{R}$

b) Maximum value of $-\frac{1}{2}e^x$ is 0 therefore — Range $f^{-1}(x) \; < 2$

c) Recognise asymtote at y=2 and plot. When $f^{-1}(x) = 0 \; then$ — $2 - \frac{1}{2}e^x = 0 \quad 2 = \frac{1}{2}e^x \quad e^x = 4 \quad x = ln4$

When x=0 — $f^{-2}(x) = 2 - \frac{1}{2}e^0 = \frac{3}{2}$

d) Simply put the values into the equation

$x_1 = -\frac{1}{2}e^{-0.3} = -0.3704 \text{ (4.d.p)}$

$x_2 = -\frac{1}{2}e^{-0.3704} = -0.3452\text{(4.d.p)}$

e) Keep going until it is stable to 3.d.p

$x_3 = -\frac{1}{2}e^{-0.3453} = -0.354\,03019$

$x_4 = -0.350\,926\,88 \; \ldots$

$x_5 = -0.352\,017\,61 \; \ldots$

$x_6 = -0.351633\,86 \; \ldots$

Therefore — $k \approx -0.352 \; (3.d.p)$

7.

$$\boldsymbol{f(x) = x^4 - 4x - 8}$$

(a) Show that there is a root of f (*x*) = 0 in the interval [–2, –1].

(3)

(b) Find the coordinates of the turning point on the graph of *y* = f(*x*).

(3)

(c) Given that $f\,(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, *a*, *b* and *c*.

(3)

(d) In the space provided on page 21, sketch the graph of *y* = f(*x*).

(3)

(e) Hence sketch the graph of *y* = |f (*x*)|.

(1)

7a) Put in -2 and then -1 and show there is a change of sign

$f(-2) = -2^4 - 4(-2) - 8 = 16 > 0$

$f(-1) = -1^4 - 4(-1) - 8 = -3 < 0$

Therefore there is a change of sign and a root between the two.

b) At turning point $\frac{dy}{dx} = 0$. Using nx^{n-1} $\qquad \frac{dy}{dx} = 4x^3 - 4 = 0$

$4x^3 = 4 \qquad x^3 = 1 \qquad x = 1$

When x=1 find y $\qquad y = 1^4 - 4(1) - 8 = -11$

c) Find (x-2) as a factor
Subtract each line

$$\begin{array}{r} \underline{x^3 + 2x^2 + 4x + 4} \\ (x-2)\surd(x^4 + 0x^3 + 0x^2 - 4x - 8) \\ \underline{x^4 - 2x^3} \\ 2x^3 + 0x^2 - 4x - 8 \\ \underline{2x^3 - 4x^2} \\ 4x^2 - 4x - 8 \\ \underline{4x^2 - 8x} \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \end{array}$$

Therefore $\qquad$ a=2, b=4, c=4

d) Graph question.

8. (i) Prove that

$$\boldsymbol{sec^2x - cosec^2x \equiv tan^2x - cot^2x}$$

(3)

(ii) Given that

$$\boldsymbol{y = arccosx, -1 \le x \le 1 \ and \ 0 \le y \le \pi}$$

(a) express $\boldsymbol{arcsin\,x}$ in terms of $\boldsymbol{y}$.

(2)

(b) Hence evaluate $\boldsymbol{arccos\,x + arcsin\,x}$. Give your answer in terms of π.

(1)

8a) Work on RHS $\qquad sec^2x - cosec^2x \equiv tan^2x - cot^2x$

Using $sec^2x = 1 + tan^2x$ $\qquad RHS = sec^2x - 1 - cot^2x$

Using $cosec^2x = 1 + cot^2x$ $\qquad RHS = sec^2x - 1 - cosec^2x + 1 = LHS$

b) $\qquad y = arccosx \qquad x = cosy = \sin(\frac{\pi}{2} - y)$

Find $m = arcsin\,x$ $\qquad m = \frac{\pi}{2} - y$

c) $$arccos\,x\; +\; arcsin\,x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$$