1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2-1)} = (ax^2 + bx + c) + \frac{dx + e}{(x^2-1)}$$

Find the values of a, b, c, d and e. **(4)**

1
Work on RHS

$$= \frac{(ax^2+bx+c)(x^2-1) + dx + e}{(x^2-1)}$$
$$= \frac{ax^4 + bx^3 + cx^2 - ax^2 - bx - c + dx + e}{(x^2-1)}$$

Compare terms on LHS and RHS

(x^4) $2 = a$

(x^3) $0 = b$

(x^2) $-3 = c - a$ therefore $c = -1$

(x) $1 = -b + d$ therefore $d = 1$

(Nos) $1 = -c + e$ therefore $e = 0$

2. A curve C has equation

$$y = e^{2x}\tan x, \quad x \neq (2n+1)\frac{\pi}{2}$$

a) Show that the turning points on C occur when $\tan x = -1$ **(6)**

b) Find an equation for the tangent to C at the point where *x=0*. **(2)**

2. a) Turning points occur when $\frac{dy}{dx} = 0$. Differentiate by parts using

$$u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = e^{2x}\frac{d(\tan x)}{dx} + \tan x.2e^{2x} = 0$$

$$\frac{dy}{dx} = e^{2x}\sec^2 x + \tan x.2e^{2x} = 0$$

Using $\sec^2 x = 1 + \tan^2 x$ $\qquad e^{2x}(\tan^2 x + 2\tan x + 1) = 0$

But $e^{2x} \neq 0$ so let $u = \tan x$ $\qquad u^2 + 2u + 1 = 0$

$(u+1)^2 = 0$ and $u = -1$ and $\tan x = -1$

b. At tangent $\frac{dy}{dx} = m$ $\qquad y = mx + c$

At *x=0*

$$m = \frac{dy}{dx} = e^0(\tan 0 + 1)^2 = 1$$

$$y = x + c$$

When $x = 0, y = e^0(tan0) = 0$ so $c=0$. Therefore $y = x$

3.

$$f(x) = ln\,(x + 2) - x + 1, \quad x > -2, x \in \mathbb{R}$$

a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$. (2)

b) Use the iterative formula

$$x_{n+1} = ln(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places. (3)

c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places. (2)

3. a For a root there will be a change of sign.

$$f(2) = \ln(4) - 2 + 1 = 0.3863 \quad > 0$$

$$f(3) = \ln(5) - 3 + 1 = -0.3906 \quad < 0$$

Therefore there is a root between 2 and 3.

b. Put in the values for x_0, x_1, x_2

$$x_1 = \ln(2.5 + 2) + 1 = 2.50408 \quad (5.d.p)$$

$$x_2 = \ln(2.50408 + 2) + 1 = 2.50498 \quad (5.d.p)$$

$$x_3 = \ln(2.50498 + 2) + 1 = 2.50518 \quad (5.d.p)$$

c. Select reasonable values either side to show change of sign.

$$f(2.5045) = \ln(4.5045) - 2.5045 + 1 = 5.7 \times 10^{-4} > 0$$

$$f(2.5055) = \ln(4.5055) - 2.5055 + 1 = -2.01 \times 10^{-4} < 0$$

Therefore there is a root of 2.505 to 3 decimal places.

4. Graph question

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}$$

Where R is the number of atoms at time *t* and c is a positive constant.

a) Find the number of atoms when the substance started to decay. (1)

It takes 5730 years for half of the substance to decay.

b) Find the value of c to 3 significant figures. (4)

c) Calculate the number of atoms that will be left when *t*=22920. (2)

d) In the spaces provided sketch the graph of R against t. (2)

a) At the start t=0, therefore $R = 1000e^{0} = 1000$

b) Half the substance is when R=500 $500 = 1000e^{-5730c}$

Therefore $\frac{1}{2} = e^{-5730c}$

Taking natural logs $\ln\left(\frac{1}{2}\right) = -5730c$

$c = -\frac{1}{5730}\ln\left(\frac{1}{2}\right) = 1.21 \times 10^{-4}(3.s.f)$

c) Put in values for c and t=22920 and work out R $R = 1000e^{-1.21\times10^{-4}.22920} = 62.5$ atoms

d) Graph

6. a) Use the double angle formula and the identity

$$\cos(A+B) \equiv cosAcosB - sinAsinB$$

To obtain an expression for $cos3x$in terms of powers of $cosx$ only. (4)

b) i) Prove that

$$\frac{cosx}{1+sinx} + \frac{1+sinx}{cosx} \equiv 2secx, \qquad x \neq (2n+1)\frac{\pi}{2}$$

(4)

ii) Hence find, for $0 < x < 2\pi$, all solutions of

$$\frac{cosx}{1+sinx} + \frac{1+sinx}{cosx} = 4$$

(3)

a) Let A=2x and B=x in formula $\cos(2x+x) \equiv cos2xcosx - sin2xsinx$

Using $cos2x = 2cos^2x - 1$ and $sin2x = 2sinxcosx$ $= (2cos^2x - 1)cosx - 2sinxcosxsinx$

$= 2cos^3x - cosx - 2sin^2xcosx$

Using $cos^2x + sin^2x = 1$ $= 2cos^3x - cosx - 2(1 - cos^2x)cosx$

$= 4cos^3x - 3cosx$

b) (i) Find a common denominator

$$\frac{cos^2x + (1+sinx)^2}{(1+sinx)cosx} = \frac{cos^2x + 1 + 2sinx + sin^2x}{(1+sinx)cosx}$$

But

$cos^2x + sin^2x = 1$

$$= \frac{2+2sinx}{(1+sinx)cosx} = \frac{2(1+sinx)}{(1+sinx)cosx} = \frac{2}{cosx} = 2secx$$

(ii) Using part i) then

$2secx = 4$

$$cosx = \frac{2}{4} = \frac{1}{2}$$

Looking at graph there are two values of $cosx$ in this interval.

$$x_1 = cos^{-1}(0.5) = \frac{\pi}{3}$$

$$x_2 = 2\pi - x_1 = \frac{5\pi}{3}$$

7. A Curve C has equation

$$\boldsymbol{y = 3sin2x + 4cos2x, \quad -\pi < x < \pi.}$$

The point A(0,4) lies on C.

a) Find an equation of the normal to the curve C at A. (5)

b) Express y in the form $Rsin(2x+\alpha)$, where R$>$ 0 and 0 $< \alpha < \frac{\pi}{2}$

Give the value of α to 3 significant figures. (4)

c) Find the coordinates of the point of intersection of the curve C with the x-axis. Give your answer to 2 decimal places. (4)

7a. For a normal $\frac{dy}{dx} = -\frac{1}{m}$

When $y = mx + c$

$$\frac{dy}{dx} = 6cos2x - 8sin2x$$

When x=0, $\frac{dy}{dx} = 6 = -\frac{1}{m}$

$$y = -\frac{1}{6}x + c$$

Sub in A(0,4)

$$4 = -\frac{1}{6}(0) + c$$

c =4 and therefore

$$y = -\frac{1}{6}x + 4$$

b. If $Rsin(2x+\alpha)$

$R = \sqrt{(3^2+4^2)}$=5

$$tan\alpha = \frac{4}{3}$$

$\alpha \approx 0.927$

c. At x-axis y=0

$$3sin2x + 4cos2x = 0$$

$$5sin(2x + 0.927) = 0$$

$$sin(2x + 0.927) = 0$$

Et $y = 2x$

$$\sin(y + 0.927) = 0, \quad -2\pi < y < 2\pi$$

$y + 0.927 = -2\pi, -\pi, 0, \pi, 2\pi$

$y = -4.069, -0.927, 2.215, 5.356$

$x = -2.03, -0.46, 1.11, 2.68 \qquad 2.d.p$

8. The functions f and g are defined as

$$\boldsymbol{f: x \mapsto 1 - 2x^3, \quad x \epsilon \mathbb{R}}$$

$$\boldsymbol{g: x \mapsto \frac{3}{x} - 4, \;\; x > 0, \;\; x \epsilon \mathbb{R}}$$

a) Find the inverse function f^{-1} **(2)**

b) Show that the composite function *gf* is

$$\boldsymbol{gf: x \mapsto \frac{8x^3 - 1}{1 - 2x^3}}$$

(4)

c) Solve $gf(x) = 0$ **(2)**

d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

a) Reverse y and x

$y = 1 - 2x^3$

$x = 1 - 2y^3$

$x = 1 - 2y^3$

$\frac{1}{2}(1 - x) = y^3$

$\sqrt[3]{\frac{1}{2}(1 - x)} = y$

b) Sub f into g

$gf: x \mapsto \frac{3}{(1 - 2x^3)} - 4$

Therefore

$gf: x \mapsto \frac{3 - 4(1 - 2x^3)}{(1 - 2x^3)} = \frac{8x^3 - 1}{1 - 2x^3}$

c)

$8x^3 - 1 = 0$

$8x^3 = 1$

$x^3 = \frac{1}{8}$ and $x = \frac{1}{2}$

d) Stationary at $\frac{dy}{dx} = 0$.

Differentiate using quotient rule

$u = 8x^3 - 1 \qquad v = 1 - 2x^3$

$\frac{du}{dx} = 24x^2 \qquad \frac{dv}{dx} = -6x^2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Therefore

$\frac{dy}{dx} = \frac{(1 - 2x^3).24x^2 - (8x^3 - 1).-6x^2}{(1 - 2x^3)^2} = 0$

Therefore

$24x^2 - 48x^5 + 48x^5 - 6x^2 = 0$

$18x^2 = 0$ and $x = 0$.

At $x = 0,\ y = \frac{0-1}{1-0} = -1$

Therefore Stationary point (0,-1)