1. Differentiate with respect to *x*, giving your answer in its simplest form,

(a) $x^2\ln(3x)$ **(4)**

(b) $\frac{sin4x}{x^3}$ **(5)**

1. a Use differentiation by parts $u\frac{dv}{dx}+v\frac{du}{dx}$

$$x^2\ln(3x) = x^2 3\times\frac{1}{3x} + \ln(3x)\times 2x$$

$$= x + 2xln(3x)$$

2. Use the quotient rule $\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$$=\frac{x^3 4cos4x - sin4x3x^2}{x^6}$$

$$=\frac{4xcos4x - 3sin4x}{x^4}$$

2. Graph question

3. The area, A mm², of a bacterial culture growing in milk, *t* hours after midday, is given by

$$A = 20e^{1.5t}, \qquad t \geq 0$$

a) Write down the area of the culture at midday. (1)

b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

a) At midday t=0 $A = 20e^0$ = 20

b) At twice the area A=40 $40 = 20e^{1.5t}$

$$2 = e^{1.5t}$$

Taking natural logs $ln\ 2 = 1.5t$

$$\frac{2}{3}ln\ 2 = t$$

$$t = 0.462\ hrs = 27.73\ mins = 28\ mins$$

4. The point *P* is the point on the curve $x = 2\ tan(y+\frac{\pi}{12})$ with *y*-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P. (7)

The gradient of the normal (m) is given by $\frac{dy}{dx} = -\frac{1}{m}$ | $x = 2\,tan(y + \frac{\pi}{12})$

Therefore $m = -\frac{dx}{dy}$ | $\frac{dx}{dy} = 2\,sec^2(y + \frac{\pi}{12})$

When $y = \frac{\pi}{4}$ | $\frac{dx}{dy} = 2\,sec^2(\frac{\pi}{4} + \frac{\pi}{12})$

$$\frac{dx}{dy} = 2\,sec^2\left(\frac{\pi}{3}\right) = \frac{2}{cos^2\left(\frac{\pi}{3}\right)} = 8$$

Therefore $y = -8x + c$

When $y = \frac{\pi}{4}$ $\quad x = 2\tan\frac{\pi}{3} = 2\sqrt{3}$

$$\frac{\pi}{4} = -8.2\sqrt{3} + c$$

$$\frac{\pi}{4} + 16\sqrt{3} = c$$

Therefore

$$y = -8x + \frac{\pi}{4} + 16\sqrt{3}$$

5. **Solve, for $0 \leq \theta \leq 180°$**

$$\mathbf{2cot^2 3\theta = 7cosec\ 3\theta - 5}$$

Give your answers in degrees to 1 decimal place.

Recognise that you need to turn cot into cosec using | $2cosec^2 3\theta - 2 = 7cosec\ 3\theta - 5$

$$cot^2\theta = cosec^2\theta - 1$$

$$2cosec^2 3\theta - 7cosec\ 3\theta + 3 = 0$$

Let $u = cosec\ 3\theta$ | $2u^2 - 7u + 3 = 0$

$$(2u - 1)(u - 3) = 0$$

So u=3 or u=0.5 | and $cosec\ 3\theta = 3$ or $cosec\ 3\theta = 0.5$

So $\sin 3\theta = \frac{1}{3}$ or $\sin 3\theta = 2$

But $\sin 3\theta \neq 2$
Let $y = 3\theta$

Then range of y is $0 \leq \theta \leq 540°$.
$y_1 = sin^{-1}\left(\frac{1}{3}\right) = 19.5$ (1.d.p)
$y_2 = 180 - 19.5$ =160.5

$$y_3 = 360 + 19.5 = 379.5$$
$$y_4 = 540 - 19.5 = 520.5$$

Therefore

$$\theta_1 = \frac{19.5}{3} = 6.5°$$
$$\theta_2 = \frac{160.5}{3} = 53.5°$$
$$\theta_3 = \frac{379.5}{3} = 126.5°$$
$$\theta_4 = \frac{520.5}{3} = 173.5°$$

6. **For**

$$f(x) = x^2 - 3x + 2cos\left(\frac{1}{2}x\right), \quad 0 \le x \le \pi$$

a) **Show that the equation *f(x)=0* has a solution in the interval** $0.8 \le x \le 0.9$ **(2)**

The curve with equation *y=f(x)* has a minimum point P

b) **Show that the x-coordinate of P is the solution of the equation**

$$x = \frac{3 + sin\left(\frac{1}{2}x\right)}{2}$$ **(4)**

c) **Using the iteration formula** $x_{n+1} = \frac{3 + sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_o = 2$

Find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places. **(3)**

d) **By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.**

a) Put in 0.8 and 0.9 and show that there is a change of sign.

$$f(0.8) = (0.8)^2 - 3(0.8) + 2cos(0.4) = 0.0821 > 0$$

Put calculator into radians.

$$f(0.9) = (0.9)^2 - 3(0.9) + 2cos(0.45) = -0.0891 < 0$$

b) *f(x)* has a minimum at $\frac{dy}{dx} = 0$.
Using nx^{n-1}

$$\frac{dy}{dx} = 2x - 3 - 2.\frac{1}{2}sin\frac{1}{2}x = 0$$

$$\frac{dy}{dx} = 2x - 3 - sin\frac{1}{2}x = 0$$

So $2x = 3 + sin\frac{1}{2}x$

Therefore $$x = \frac{1}{2}(3 + sin\frac{1}{2}x)$$

c) $$x_{n+1} = \frac{1}{2}\left(3 + sin\frac{1}{2}x_n\right) \quad x_o = 2$$

$$x_1 = \frac{1}{2}(3 + \sin 1) = 1.9207 = 1.921\ (3.d.p)$$

$$x_2 = \frac{1}{2}\left(3 + \sin\frac{1.9207}{2}\right) = 1.9097 = 1.910\ (3.d.p)$$

$$x_3 = \frac{1}{2}\left(3 + \sin\frac{1.9097}{2}\right) = 1.908\ (3.d.p)$$

d) Choose an interval $1.90775 < x < 1.90785$ and substitute back into *f′(x)*.

$$f'(x) = 2x - 3 - \sin(\frac{x}{2})$$

$$f'(1.90775) = -1.63 \times 10^{-4}$$

$$f'(1.90785) = 7.66 \times 10^{-6}$$

As there is a change of sign there is a root between these two.

7. The function f is defined by

$$f: x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4} \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

a) Show that $f(x) = \frac{1}{2x-1}$ **(4)**

b) Find *f′(x)* **(3)**

a. Find the domain of f^{-1}

(1)

$$g(x) = ln(x+1)$$

c) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e.

(4)

7.

a. Therefore $$f(x) = \frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{x+4}$$

Find common denominator $$f(x) = \frac{3(x+1) - (2x-1)}{(2x-1)(x+4)}$$

$$f(x) = \frac{x+4}{(2x-1)(x+4)}$$

$$f(x) = \frac{1}{(2x-1)}$$

b. Replace f(x) with x and x with y and rearrange. $$x = \frac{1}{(2y-1)}$$

$$2y - 1 = \frac{1}{x}$$

Therefore $$y = \frac{1}{2}(\frac{1}{x} + 1)$$

c. $$x > 0$$

d. $$fg(x) = \frac{1}{2\ln(x+1) - 1} = \frac{1}{7}$$

$$2\ln(x+1) - 1 = 7$$

$$\ln(x+1) = 4$$

$$x + 1 = e^4$$

$$x = e^4 - 1$$

8. a) Starting from the formulae for $sin(A+B)$ and $cos(A+B)$, prove that

$$\boldsymbol{tan(A+B) = \frac{tan\,A + tanB}{1 - tanAtanB}}$$

(4)

b. Deduce that

$$\boldsymbol{tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}tan\,\theta}{\sqrt{3} - tan\theta}}$$

(3)

c. Hence, or otherwise, solve for $0 \leq \theta \leq \pi$.

$$\boldsymbol{1 + \sqrt{3tan\theta} = \left(\sqrt{3} - tan\theta\right)tan(\pi - \theta)}$$

(6)

a) $$\sin(A+B) = sinA\,cosB + cosAsin\,B$$

$$\cos(A+B) = \cos A\,cosB - sinAsinB$$

Therefore $$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$$

Divide by cos A cos B $$= \frac{\frac{sinA}{cosA} - \frac{sinB}{cosB}}{1 - \frac{\sin A}{\cos A}\frac{\sin B}{\cos B}} = \frac{\tan A - tanB}{1 - tanAtanB}$$

b) Fill in A+B formula

$$=\frac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}} = \frac{tan\theta + \frac{\sqrt{3}}{3}}{1 - tan\theta\frac{\sqrt{3}}{3}}$$

x by $\sqrt{3}$

$$=\frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$$

c) Rearrange

$$\frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta} = \tan(\pi - \theta)$$

Therefore

$$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$$

So

$\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$ and $\theta = \frac{5\pi}{12}$

Or

$\left(\theta + \frac{\pi}{6}\right) = (2\pi - \theta)$ and $\theta = \frac{11\pi}{12}$