1. Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the *x*-axis at the point *A* where *x* = *α*.

To find an approximation to *α*, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.

Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that *α* = 2.359 correct to 3 decimal places.

(3)

1 a) Put in x_0 and so on

$$x_1 = \frac{2}{(2.5)^2} + 2 = 2.320\ (3.d.p)$$

$$x_2 = \frac{2}{(2.32)^2} + 2 = 2.372\ (3.d.p)$$

$$x_3 = \frac{2}{(2.372)^2} + 2 = 2.356\ (3.d.p)$$

$$x_4 = \frac{2}{(2.356)^2} + 2 = 2.360\ (3.d.p)$$

b) Choose values of α either side

$$2.3585 < \propto < 2.3595$$

$$y = -x^3 + 2x^2 + 2 = -2.3585^3 + 2 \times 2.3585^2 + 2$$
$$y = 5.84 \times 10^{-3}$$
$$y = -x^3 + 2x^2 + 2 = -2.3595^3 + 2 \times 2.3595^2 + 2$$
$$y = -1.42 \times 10^{-3}$$

The change of sign proves that there is a root between these values at α=2.359

2. (a) Use the identity

$$\cos^2 x + \sin^2 x = 1$$

to prove that

$$\tan^2 x = \sec^2 x - 1$$

(2)

(b) Solve, for $0 \le \theta < 360°$ the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

(6)

2. a)	$cos^2x + sin^2x = 1$
	$sin^2x = 1 - cos^2x$
Divide by cos^2x	$\frac{sin^2x}{cos^2x} = \frac{1}{cos^2x} - \frac{cos^2x}{cos^2x}$
Therefore	$tan^2x = sec^2x - 1$
b) Recognise that you need to get rid of tan using the formula just proved.	$2(sec^2\theta - 1) + 4sec\theta + sec^2\theta = 2$
	$3sec^2\theta + 4sec\theta - 4 = 0$
Let $sec\theta = u$	$3u^2 + 4u - 4 = 0$
	$(3u - 2\ )(u + 2) = 0 \quad u = \frac{2}{3}\ or\ u = -2$
So	$sec\theta = \frac{2}{3}\ or\ sec\theta = -2 \quad \frac{1}{cos\theta} = \frac{2}{3}\ \ or\ \frac{1}{cos\theta} = -2$
	$cos\theta = \frac{3}{2}\ but > 1\ \ so\ cos\theta = -\frac{1}{2} \quad \theta_1 = 120°$
By examining the cos curve we see that there is another values at 360-θ_1	Therefore $\theta = 120\ or\ 240°$

3. Rabbits were introduced onto an island. The number of rabbits, *P*, *t* years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\frac{dP}{dt}$

(2)

(d) Find *P* when $\frac{dP}{dt} = 50$

(3)

3. a) When t=0 then	$P_o = 80$
b)	$1000 < 80e^{\frac{1}{5}t}$
	$12.5 < e^{\frac{1}{5}t}$

$ln12.5 < \frac{1}{5}t$

$5ln12.5 < t \quad t > 12.62 \quad therefore\ t = 13$ years

c) Using $\frac{d(e^x)}{dx} = e^x$

$\frac{dP}{dt} = 80\frac{1}{5}e^{\frac{1}{5}t} = 16e^{\frac{1}{5}t}$

d)

$\frac{dP}{dt} = 16e^{\frac{1}{5}t} = 50$

$e^{\frac{1}{5}t} = 3.125$

$\frac{1}{5}t = \ln 3.125 \quad t = 5ln3.125$

Put back into $P = 80e^{\frac{1}{5}t}$ To find P

$P = 80e^{\frac{1}{5}5ln3.125} = 80e^{ln3.125} = 80 \times 3.125$
$= 250\ rabbits.$

4. (i) Differentiate with respect to *x*

(a) x^2cos3x **(3)**

(b) $\frac{\ln(x^2+1)}{x^2+1}$ **(4)**

(ii) A curve *C* has the equation

$$y = \sqrt{4x+1}, \qquad x > -\frac{1}{4}, \quad y > 0$$

The point *P* on the curve has *x*-coordinate 2. Find an equation of the tangent to *C* at *P* in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.

(6)

4 (i) a) Differentiate by parts $u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2 \quad \frac{du}{dx} = 2x \quad v = cos3x \quad \frac{dv}{dx} = -3sin3x$

$= -3x^2sin3x + 2xcos3x$

b) Use substitution let $m = x^2 + 1 \quad \frac{dm}{dx} = 2x$

Using quotient rule $= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{d}{dx} = \frac{dm}{dx}.\frac{d}{dm} = 2x\frac{d}{dm}(\frac{\ln m}{m})$

$u = \ln m \quad \frac{du}{dm} = \frac{1}{m} \qquad v = m \quad \frac{dv}{dm} = 1$

$= \frac{m\frac{1}{m} - lnm}{m^2} = \frac{1 - lnm}{m^2}$

$$\frac{d}{dx} = 2x\left(\frac{1 - lnm}{m^2}\right) = 2x\left(\frac{1 - \ln(x^2+1)}{(x^2+1)^2}\right)$$

c) Using $\frac{dy}{dx} = m$

$y = \sqrt{4x+1}$ $\quad \frac{dy}{dx} = \frac{1}{2} \times 4 \times (4x+1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = 2(4x+1)^{-\frac{1}{2}}$ $\quad$ $when\ x = 2, \frac{dy}{dx} = \frac{2}{3}$

Therefore $\quad y = \frac{2}{3}x + c$

Find y when x=2 $\quad y = \sqrt{4x+1} = \sqrt{9} = 3$

Fill in (2,3) to find c $\quad 3 = \frac{4}{3} + c \quad c = \frac{5}{3}$

Therefore $\quad y = \frac{2}{3}x + \frac{5}{3}$

x3 and rearrange $\quad 3y = 2x + 5 \quad 0 = 2x - 3y + 5$

5. Graph Question

6. (a) Use the identity

$$\cos(A + B) = \cos A\, cosB - sinAsinB$$

, to show that

$$cos2A = 1 - 2sin^2A$$

(2)

The curves *C*1 and *C*2 have equations

$$C1: y = 3sin\,2x$$
$$C2: y = 4\,sin^2 x - 2\,cos\,2x$$

(b) Show that the *x*-coordinates of the points where *C*1 and *C*2 intersect satisfy the equation

$$4cos\,2x + 3sin\,2x = 2$$

(3)

(c) Express $4cos\,2x + 3sin\,2x$ in the form $R\,cos\,(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \le x < 180°$ all the solutions of

$$4cos\,2x + 3sin\,2x = 2$$

giving your answers to 1 decimal place.

(4)

6 a) Let B=A

$\cos(A + A) = \cos A\, cosA - sinAsinA$

$\cos 2A = \cos^2 A\ - sin^2 A$

Using
$cos^2 x + sin^2 x = 1$

$\cos 2A = (1 - sin^2 A)\ - sin^2 A$

$\cos 2A = 1 - 2sin^2 A$

b) $C1: y\ =\ 3sin\ 2x$
$C2: y\ =\ 4\ sin^2\ x\ -\ 2\ cos\ 2x$

$3sin2x = 4sin^2\ x\ -\ 2\ cos\ 2x$

Using formula proved in a)

$3sin2x = 4\left(\frac{1}{2}\right)(1 - cos2x) -\ 2\cos 2x$

$3sin2x = (2 - 2cos2x) -\ 2\cos 2x$

$4cos2x + 3sin2x = 2$

c) Compare with expansion and then compare sin2x and cos2x

$3sin\ 2x\ +\ 4cos2x\ =\ Rcos(2x\ - \alpha\ )$
$3sin\ 2x\ +\ 4cos2x\ =\ Rcos2x\ cos\alpha\ +\ Rsin\ 2xsin\alpha$
$3 =\ \ Rsin\alpha$
$4 =\ Rcos\alpha$

Using
$cos^2 x + sin^2 x = 1$

$R^2 = 3^2 + 4^2 \qquad R = 5$
$tan\ \propto = \frac{3}{4}$

Therefore

$4cos2x + 3sin2x =\ 5cos(2x\ -\ 36.87)$

d) Using part c)

$5cos(2x\ -\ 36.87) = 2$

$cos(2x\ -\ 36.87) = \frac{2}{5}$
$(2x\ -\ 36.87)\ =\ 66.42°\ (2.d.p)$
$(2x\ -\ 36.87)\ =\ 360\ -\ 66.42\ = 293.58°\ (2.d.p)$
$x\ =\ \frac{1}{2}(66.42 + 36.87)\ = 65.1\ (1.d.p)$
$x\ = \frac{1}{2}(293.58 + 36.87)\ = 165.2\ (1.d.p)$

7. The function f is defined by

$$\boldsymbol{f(x) = 1 - \frac{2}{x+4} + \frac{x-8}{(x-2)(x+4)},\quad x \in \mathbb{R}, x \neq -4, x \neq 2}$$

(a) Show that

$$\boldsymbol{f(x) = \frac{x-3}{x-2}}$$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq ln2$$

(b) Differentiate g(*x*) to show that

$$g'(x) = \frac{e^x}{(e^x - 2)^2}$$

(3)

(c) Find the exact values of *x* for which g′(*x*) = 1

(4)

$$f(x) = 1 - \frac{2}{x+4} + \frac{x-8}{(x-2)(x+4)}$$
$$f(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$$
$$f(x) = \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$$
$$f(x) = \frac{x^2 + x - 12}{(x-2)(x+4)} = \frac{(x-3)(x+4)}{(x-2)(x+4)} = \frac{x-3}{x-2}$$

b) Differentiate using quotient rule

$$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$u = e^x - 3 \quad \frac{du}{dx} = e^x \qquad v = e^x - 2 \quad \frac{dv}{dx} = e^x$$

$$= \frac{(e^x - 2)e^x - (e^x - 3)e^x}{(e^x - 2)^2} = \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$$
$$= \frac{e^x}{(e^x - 2)^2}$$

c)

$$\frac{e^x}{(e^x - 2)^2} = 1$$
$$e^x = (e^{2x} - 4e^x + 4)$$

$$0 = e^{2x} - 5e^x + 4$$

Let $e^x = u$ $\qquad 0 = u^2 - 5u + 4 \qquad 0 = (u-4)(u-1)$

Therefore

$e^x = 1 \qquad x = 0$

$e^x = 4 \qquad x = ln4$

8. (a) Write down $sin\ 2x$ in terms of $sin\ x$ and $cos\ x$.

(1)

(b) Find, for 0 < *x* < *π*, all the solutions of the equation

$$\boldsymbol{cosec\, x - 8cos\, x = 0}$$

giving your answers to 2 decimal places.

(5)

a) $sin2x = 2sinxcosx$

b) Using $cosecx = \frac{1}{sinx}$

$$\frac{1}{sinx} - 8cos\, x = 0$$

$$\frac{1}{sinx} = 8\cos x$$

$$1 = 8\cos x\, \text{sinx}$$

Using results from part a)

$$1 = 4\text{sin2x}$$

$$\text{sin2x} = \frac{1}{4}$$

Let y=2x Range $0 < y < 2\pi$

$$\text{y} = \sin^{-1}\frac{1}{4} \quad y_1 = 0.25\ (2.d.p)$$

For sin $y_2 = \pi - 0.25 = 2.89$

$$2\text{x} = 0.25, 2.89$$

$$\text{x} = 0.13, 1.45$$