1. Find $\int(3x^2 + 4x^5 - 7)dx$

(4)

1. Using $\frac{1}{n+1}x^{n+1}$

$$= \frac{3}{3}x^3 + \frac{4}{6}x^6 - 7x + c$$

$$= x^3 + \frac{2}{3}x^6 - 7x + c$$

2. (a) Write down the value of $16^{\frac{1}{4}}$

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$

(2)

a) $= \sqrt[4]{16} = 2$

b) $= 16^{\frac{3}{4}}(x^{12})^{\frac{3}{4}} = 2^3x^{\frac{36}{4}} = 8x^9$

3. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and be are integers.

(4)

3. Multiple by $\frac{2-\sqrt{3}}{2-\sqrt{3}}$ to remove surd from base.

$$= \frac{5-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-2\sqrt{3}-5\sqrt{3}+3}{4-2\sqrt{3}+2\sqrt{3}-3}$$

$$= \frac{13-7\sqrt{3}}{1} = 13 - 7\sqrt{3}$$

4. The point A(-6,4) and the point B(8,-3) lie on the line L.

a) Find and equation for the line L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

4. Find the gradient by $m = \frac{y_2-y_1}{x_2-x_1}$
Therefore

$$m = \frac{-3-4}{8--6} = -\frac{7}{14} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

Sub in P(-6,4)

$$4 = -\frac{1}{2}(-6) + c$$

$$4 = 3 + c \qquad c = 1$$

Therefore

$$y = -\frac{1}{2}x + 1$$

Multiply by 2

$$2y = -x + 2$$

$$x + 2y - 2 = 0$$

b) The length of the line is given by

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore

$$= \sqrt{(8 - -6)^2 + (-3 - 4)^2}$$

$$= \sqrt{14^2 + 7^2} = \sqrt{245} = \sqrt{49 \times 5}$$

$$= 7\sqrt{5} \qquad k = 7$$

5. a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that

$$y = 5x - 7 + \frac{2\sqrt{x} + 3}{x}, \qquad x > 0.$$

b) Find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

5a)

$$= \frac{2\sqrt{x} + 3}{x} = \frac{2\sqrt{x}}{x} + \frac{3}{x} = 2.x^{\frac{1}{2}}.x^{-1} + 3x^{-1}$$

$$= 2x^{-\frac{1}{2}} + 3x^{-1} \qquad p = -\frac{1}{2} \qquad q = -1$$

b) Therefore

$$y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$$

Using nx^{n-1}

$$\frac{dy}{dx} = 5 + 2.-\frac{1}{2}x^{-\frac{3}{2}} - 3x^{-2}$$

$$\frac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$$

6. Graph question

7. A sequence is given by

$$x_1 = 1$$
$$x_{n+1} = x_n(p + x_n),$$

Where p is a constant (p≠0).

a) Find x_2 in terms of p.

(1)

b) Show that $x_3 = 1 + 3p + 2p^2$ **(2)**

Given that $x_3 = 1$

c) find the value of p

(3)

d) Write down the value of x_{2008}

(2)

7a) $x_2 = (p+1)$

b) $x_3 = x_2(p + x_2) = (p+1)(p+p+1)$

$x_3 = (p+1)(2p+1) = 2p^2 + 3p + 1$

$x_3 = 1 + 3p + 2p^2$

c) $1 = 1 + 3p + 2p^2$

$3p + 2p^2 = 0 \qquad p(3+2p) = 0$

But p≠0 therefore $2p = -3 \qquad p = -\frac{3}{2}$

d) Try at few values

$x_1 = 1$

$x_2 = (p+1) = -\frac{1}{2}$

$x_3 = 1$

$x_4 = (p+1) = -\frac{1}{2}$

Therefore $x_{2008} = (p+1) = -\frac{1}{2}$

8. The equation

$$\boldsymbol{x^2 + kx + 8 = k}$$

has no real solutions for x

a) Show that k satisfies $k^2 + 4k - 32 < 0$ **(3)**

b) Hence find the set of possible values of k.

(4)

8. For no real solutions $x^2 + kx + 8 - k = 0$

$b^2 - 4ac < 0$

Re-write equation

Therefore $k^2 - 4(8-k) < 0$

$k^2 - 32 + 4k < 0$

$(k-4)(k+8) < 0$

b) Critical values at $(k-4)(k+8) = 0$ $\quad k = 4 \text{ or } k = -8$

The k^2 curve is positive so is a U shape which is <0 between critical values. $-8 < k < 4$

9. The curve C has equation $y = f(x)$, $x > 0$ and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$

Given that the point P(4,1) lies on C,

a) find f(x) and simplify your answer.

(6)

b) Find the equation of the normal to C at the point P(4,1)

(4)

9. a)

$$f'(x) = \frac{dy}{dx} = 4x - 6\sqrt{x} + \frac{8}{x^2}$$

$$\frac{dy}{dx} = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$$

Integrate to find f(x)

$$y = f(x) = \int 4x - 6x^{\frac{1}{2}} + 8x^{-2}\, dx$$

Using

$$\frac{1}{n+1}x^{n+1}$$

$$y = f(x) = \frac{4}{2}x^2 - \frac{6}{\frac{3}{2}}x^{\frac{3}{2}} + \frac{8}{-1}x^{-1} + c$$

$$f(x) = 2x^2 - 4x^{\frac{3}{2}} - 8x^{-1} + c$$

Put P into the equation to find c

$$1 = 32 - 32 - 2 + c$$

$$3 = c$$

b) For the normal

$$\frac{dy}{dx} = -\frac{1}{m}$$

$$\frac{dy}{dx} = 16 - 12 + \frac{1}{2} = \frac{9}{2}$$

Therefore

$$m = -\frac{2}{9} \qquad y = -\frac{2}{9}x + c$$

Put in values for P to find c

$$1 = -\frac{8}{9} + c \qquad c = \frac{17}{9}$$

Therefore $y = -\frac{2}{9}x + \frac{17}{9}$

Multiple by 9. $9y = -2x + 17$

10. The curve *C* has equation

$$\boldsymbol{y = (x+3)(x-1)^2}$$

(a) Sketch *C* showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of *C* can be written in the form

$$\boldsymbol{y = x^3 + x^2 - 5x + k}$$

where *k* is a positive integer, and state the value of *k*.

(2)

There are two points on *C* where the gradient of the tangent to *C* is equal to 3.

(c) Find the *x*-coordinates of these two points.

(6)

10 a) Find x when y=0 $0 = (x+3)(x-1)^2 \quad x = -3 \; and \; x = 1$

Note there is double root at x=1 which means there is a turning point. When x=0 then $y = (3)(-1)^2 = 3$
Plot standard positive x^3 curve.

b) Expand out $y = (x+3)(x^2 - 2x + 1)$

$y = x^3 + 3x^2 - 2x^2 + x - 6x + 3$

$y = x^3 + x^2 - 5x + 3 \qquad k = 3$

The gradient of the curve is $\frac{dy}{dx}$ $\frac{dy}{dx} = 3x^2 + 2x - 5 = 3$

$3x^2 + 2x - 8 = 0$

$(3x-4)(x+2) = 0$

$x = \frac{4}{3} \quad or \; x = -2$

Note that many questions ask for the coordinates not just the x- coordinate and so it would be important to plug back into the equation to find y.

11. The first term of an arithmetic sequence is 30 and the common difference is –1.5

(a) Find the value of the 25th term.

(2)

The *r* th term of the sequence is 0.
(b) Find the value of *r*.

(2)

The sum of the first *n* terms of the sequence is *Sn* .
(c) Find the largest positive value of *Sn* .

(3)

11a) Using $u_n = a + (n-1)d$

$u_{25} = 30 + 24 \times -1.5$

$u_{25} = -6$

b) $u_r = 0$

$0 = 30 + (r-1) \times -1.5$

$0 = 31.5 - 1.5r$

Multiple by 2 $\quad 0 = 63 - 3r \quad r = 21$

c) As the common difference is negative each term will get smaller after r=21 therefore the sum up to S_{21} or indeed S_{20} as u_{21} is 0.

Using $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[60 + (20-1) \times -1.5] = 315$